Extend 3-4 Geometry Lab: Equations of Perpendicular Bisectors

Find the equation of the perpendicular bisector $\overline{PQ}$ for the given endpoints.

1. $P(5, 2)$, $Q(7, 4)$

SOLUTION:

Use the Midpoint Formula to find the coordinates of the midpoint of $\overline{PQ}$.

$$M\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) = M\left(\frac{5+7}{2}, \frac{2+4}{2}\right)$$
$$= M(6,3)$$

A perpendicular bisector is perpendicular to the segment through the midpoint. In order to find the slope of the bisector, first find the slope of $\overline{PQ}$.

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{4-2}{7-5} \\ &= \frac{2}{2} \\ &= 1 \end{aligned}$$

So, the slope of the line perpendicular to $\overline{PQ}$ is –1.

Now use the point-slope form to write the equation of the line.

Here, $m = –1$ and $(x_1, y_1) = (6, 3)$.

$$y - 3 = -1(x-6).$$
$$y = -x + 9$$

The equation of the perpendicular bisector of $\overline{PQ}$ is $y = -x + 9$.

2. $P(–3, 9)$, $Q(–1, 5)$

SOLUTION:

Use the Midpoint Formula to find the coordinates of the midpoint of $\overline{PQ}$.

$$M\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) = M\left(\frac{-3+(-1)}{2}, \frac{9+5}{2}\right)$$
$$= M(-2,7)$$

A perpendicular bisector is perpendicular to the segment through the midpoint. In order to find the slope of the bisector, first find the slope of $\overline{PQ}$.

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{5-9}{-1-(-3)} \\ &= \frac{-4}{2} \\ &= -2 \end{aligned}$$

So, the slope of the line perpendicular to $\overline{PQ}$ is $\frac{1}{2}$.

Now use the point-slope form to write the equation of the line.

Here, $m = \frac{1}{2}$ and $(x_1, y_1) = (–2, 7)$.

$$y - 7 = \frac{1}{2}(x - (-2)).$$
$$y = \frac{1}{2}x + 8$$

The equation of the perpendicular bisector of $\overline{PQ}$ is $y = \frac{1}{2}x + 8$.

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3. $P(-6, -1)$, $Q(8, 7)$

SOLUTION:

Use the Midpoint Formula to find the coordinates of the midpoint of $\overline{PQ}$.

$$M\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) = M\left(\frac{-6+8}{2}, \frac{-1+7}{2}\right) = M(1, 3)$$

A perpendicular bisector is perpendicular to the segment through the midpoint. In order to find the slope of the bisector, first find the slope of $\overline{PQ}$.

$$m = \frac{y_2-y_1}{x_2-x_1} = \frac{7-(-1)}{8-(-6)} = \frac{8}{14} = \frac{4}{7}$$

So, the slope of the line perpendicular to $\overline{PQ}$ is $-\frac{7}{4}$.

Now use the point-slope form to write the equation of the line.

Here, $m = -\frac{7}{4}$ and $(x_1, y_1) = (1,3)$.

$$y-3 = -\frac{7}{4}(x-1).$$

$$y-3 = -\frac{7}{4}x+\frac{7}{4}$$

$$y = -\frac{7}{4}x+\frac{19}{4}$$

The equation of the perpendicular bisector of $\overline{PQ}$ is $y = -\frac{7}{4}x+\frac{19}{4}$.

4. $P(-2, 1)$, $Q(0, -3)$

SOLUTION:

Use the Midpoint Formula to find the coordinates of the midpoint of $\overline{PQ}$.

$$M\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) = M\left(\frac{-2+0}{2}, \frac{1+(-3)}{2}\right) = M(-1,-1)$$

A perpendicular bisector is perpendicular to the segment through the midpoint. In order to find the slope of the bisector, first find the slope of $\overline{PQ}$.

$$m = \frac{y_2-y_1}{x_2-x_1} = \frac{-3-1}{0-(-2)} = \frac{-4}{2} = -2$$

So, the slope of the line perpendicular to $\overline{PQ}$ is $\frac{1}{2}$.

Now use the point-slope form to write the equation of the line.

Here, $m = \frac{1}{2}$ and $(x_1, y_1) = (-1,-1)$.

$$y-(-1) = \frac{1}{2}(x-(-1)).$$

$$y+1 = \frac{1}{2}x+\frac{1}{2}$$

$$y = \frac{1}{2}x-\frac{1}{2}$$

The equation of the perpendicular bisector of $\overline{PQ}$ is $y = \frac{1}{2}x-\frac{1}{2}$.

5. $P(0, 1.6)$, $Q(0.5, 2.1)$

SOLUTION:

Use the Midpoint Formula to find the coordinates of the midpoint of $\overline{PQ}$.

$$M\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)=M\left(\frac{0+0.5}{2},\frac{1.6+2.1}{2}\right)$$
$$=M(0.25,1.85)$$

A perpendicular bisector is perpendicular to the segment through the midpoint. In order to find the slope of the bisector, first find the slope of $\overline{PQ}$.

$$m=\frac{y_2-y_1}{x_2-x_1}=\frac{2.1-1.6}{0.5-0}=\frac{0.5}{0.5}=1$$

So, the slope of the line perpendicular to $\overline{PQ}$ is –1.

Now use the point-slope form to write the equation of the line.

Here, $m=-1$ and $(x_1,y_1)=(0.25,1.85)$.

$$y-1.85=-1(x-0.25).$$
$$y-1.85=-x+0.25$$
$$y=-x+2.1$$

6. $P(–7, 3)$, $Q(5, 3)$

SOLUTION:

Use the Midpoint Formula to find the coordinates of the midpoint of $\overline{PQ}$.

$$M\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)=M\left(\frac{-7+5}{2},\frac{3+3}{2}\right)$$
$$=M(-1,3)$$

A perpendicular bisector is perpendicular to the segment through the midpoint. In order to find the slope of the bisector, first find the slope of $\overline{PQ}$.

$$m=\frac{y_2-y_1}{x_2-x_1}=\frac{3-3}{5-(-7)}=\frac{0}{12}=0$$

So, the slope of the line perpendicular to $\overline{PQ}$ is undefined and hence it is a vertical line.

The *x*-coordinate of the midpoint is –1. So, equation of a vertical line through the point (–1, 3) is $x = –1$.

7. **CHALLENGE** Find the equations of the lines that contain the sides of ΔXYZ with vertices $X(–2, 0)$, $Y(1, 3)$, and $Z(3, –1)$.

SOLUTION:

Find the slopes of the sides $\overline{XY}, \overline{YZ}$, and $\overline{XZ}$.

$$m_{xy}=\frac{y_2-y_1}{x_2-x_1}=\frac{3-0}{1-(-2)}=\frac{3}{3}=1$$

$$m_{yz}=\frac{y_2-y_1}{x_2-x_1}=\frac{-1-3}{3-1}=\frac{-4}{2}=-2$$

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$$\begin{aligned} m_{xz} &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{-1-0}{3-(-2)} \\ &= \frac{-1}{5} \\ &= -\frac{1}{5} \end{aligned}$$

$m_{xy} = 1$ and $(x_1, y_1) = (–2, 0)$

So, the equation of the side $\overline{XY}$ is:

$$y - 0 = 1(x - (-2)).$$
$$y = x + 2$$

$m_{yz} = –2$ and $(x_1, y_1) = (1, 3)$.

So, the equation of the side $\overline{YZ}$ is:

$$y - 3 = -2(x - 1).$$
$$y = -2x + 5$$

$m_{xz} = -\frac{1}{5}$ and $(x_1, y_1) = (–2, 0)$.

So, the equation of the side $\overline{XZ}$ is:

$$y - 0 = -\frac{1}{5}(x - (-2)).$$
$$y = -\frac{1}{5}x - \frac{2}{5}$$