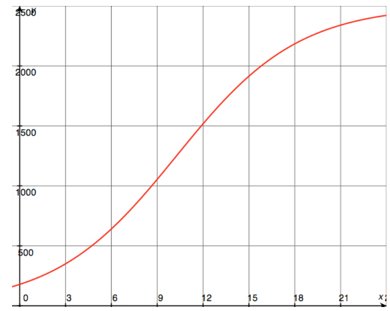
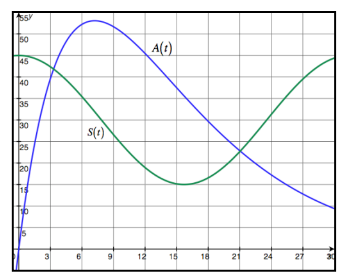
AB Calculus – Introduction

Before we get into what calculus is, here are several examples of what you could do BC (before calculus) and what you will be able to do at the end of this course.



Example 1: On April 15, many people mail in their taxes to the Internal Revenue Service. The town of Newton monitors the number of tax forms that are mailed from their post office. The total number of tax forms that are mailed from the Newton post office on April 15 is modeled by the function: $M(t)=\dfrac{2500}{1+13e^{-0.25t}}$ where t is the number of hours from 12 midnight on April 14 through 12 midnight on April 15 and $M(t)$ is the total number of letters mailed that day from the Newton post office.

What you can do with precalculus	What you will be able to do with calculus
Find the number of tax forms mailed by 9 PM on April 15. **Answer**: $M(21)\approx 2{,}340$	Find the average rate of tax forms mailed between 6 PM and 9 PM. **Answer**: 51.9 forms per hour
	Find the rate that the tax forms are coming into the post office at 9 PM. **Answer**: 37.3 forms/hour.
	Find the time of day when the letters are coming into the post office at the fastest rate and what is that rate? **Answer**: Approximately 10:15 AM at the rate of 156.25 forms/hour.
	What is the total number of hours that all the forms sit in the post office? **Answer**: 33,939 hours.



Example 2: A new car called the Sexus has its plant next to its only dealership. Cars are sold directly from the plant. Suppose at the start of the month of May there are 100 cars on the lot waiting to be sold. Cars come off the assembly line at the rate of $A(t)=20t\left(2^{-0.2t}\right)$ and cars are sold at the rate of $S(t)=30+15\cos(0.2t)$ where t represents the day of the month (for May 1, $t=0$ and for May 31, $t=30$).

What you can do with precalculus	What you will be able to do with calculus
Find the rate of cars that are produced on May 15. Answer: $A(14)=40.2$ cars/day	Find the total number of cars available to be sold in the month of May. **Answer**: Approx. 1,057 cars
Find the rate of cars that are sold on May 15. Answer: $S(14)=15.9$ cars/day	Find the total numbers of cars sold in May. **Answer**: 879 cars.
	Find the average number of cars produced per day in May. **Answer**: 31.9 cars/day.
	On what day will there be a maximum number of cars waiting to be sold and approximately what will that number be? **Answer**: May 22 and 355 cars will be on the lot.
	On what day will there be a minimum number of cars waiting to be sold and approximately what will that number be? **Answer**: May 4 and 164 cars will be on the lot.

Introductory Lesson 1

All the math courses you have ever taken have been, in a sense, precalculus. In these courses, you are analyzing numbers and expressions and determining their current state. (ex: Find the value of x^2 when $x = 4$). In calculus, you analyze how things **change**. When real-life quantities change, they will either change in a positive direction, or in a negative direction. Your first assignment is more an English one than math. You are to find words that are used to denote positive change, negative change, or no change. I have started you off with one. Find as many as you can. When stuck, think of real life areas where change occurs such as academics, sports, economics, biology, music, etc. Get your dictionaries out!

Positive Change	**Negative Change**	**No Change**
1. Increasing	1. Decreasing	1. Constant
2. Rise	2. Reduce	2. Steady
3. Expand	3. Diminish	3. Stable
4. Intensify	4. Lessen	4. Invariable
5. Augment	5. Reduce	5. Fixed
6. Strengthen	6. Decline	6. Unwavering
7. Enlarge	7. Dwindle	7. Permanent
8. Amplify	8. Shrink	8. Unchanging
9. Enhance	9. Fall off	9. Static
10. Improve	10. Cut	10. Stationary
11. Swell	11. Weaken	
12. Extend	12. Worsen	
13. Escalate	13. Contract	
14. Ascend	14. Deflate	
15. Grow	15. Descend	

Since calculus is the study of change, let's talk about change around us. Give several things that are changing about **YOU** right now.

1. Height is changing 2. Fingernails are growing 3. Hair is growing 4. Knowledge is increasing

Choose one. How do we know the change is occurring? Is the change a constant change?

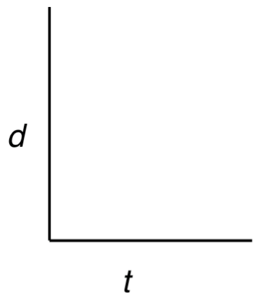
Not necessarily. A kid can stay constant for awhile and then just shoot up quickly.
With knowledge, a student can cram for a test and increase the knowdge quickly.

In calculus, we study four topics: 1) limits, 2) derivatives, 3) integrals (one kind) and 4) integrals (another kind). All of these 4 topics are related to the concept of change. Everything we do in this course will be related to these 4 concepts. Although we will be involved in many details, everything comes down to these 4 concepts. Your job in this course will be to answer the question "which of these 4 topics does the problem I am attempting to solve apply to." Although we will deal with many little details, we always need to see the big picture. These introductory lessons will focus on the **derivative** and the **definite integral** and meant to give you an idea what this fascinating course is all about.

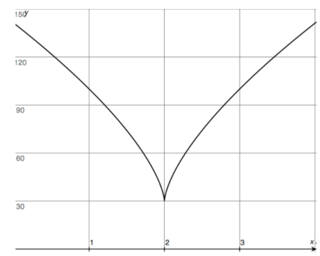
Introductory Lesson 2

1. Joan is lifting weights as shown in the picture. She starts at the leftmost position, moves to the rightmost position, and then goes back to the leftmost position. At first, the weight move slowly but once she gets the weight moving and gains some momentum, the weight moves faster. Returning to the leftmost position just reverses the movement. The entire movement takes 4 seconds. Focus on the angle in degrees of her elbows as she moves the weight. When the weight goes up, the angle decreases. When the weight goes down, the angle increases.

Make a sketch of the graph of time vs. degrees. Be sure your graph has a scale.



If d is modeled by $d = 30 + 70(x-2)^{2/3}$, make a sketch of the graph of time vs. degrees. Be sure your graph has a scale.



Make a table of values of d for each second from t = 0 to t = 4. Round to the nearest 0.1 degree.

t	0	0.4	0.8	1.2	1.6	2.0	2.4	2.8	3.2	3.6	4.0
d	141.1	125.8	109.0	90.3	68.0	30.0	68.0	90.3	109.0	125.8	141.1

At time t = 0.8 seconds, is the weight going up or down? $\boxed{\text{Going up}}$ How do you know?

$\boxed{\text{The elbow angle is getting smaller so the weight is going up.}}$

Our goal is to find the *instantaneous rate of change of the elbow angle* at time t = 1.2 second. In order to do that we will start with finding *average rate of change of the elbow angle* at time t = 1.2 second.

$$\text{Average rate of change } = \frac{d(1.2)-d(0.8)}{1.2-0.8} = \frac{90.3-109.0}{0.4} = \frac{-46.8°}{\text{sec}}$$

$$\text{Average rate of change } = \frac{d(1.6)-d(1.2)}{1.6-1.2} = \frac{68-90.3}{0.4} = \frac{-55.8°}{\text{sec}}$$

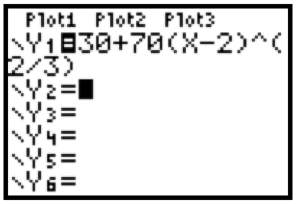
$$\text{Average rate of change } = \frac{d(1.6)-d(0.8)}{1.6-0.8} = \frac{90.3-109.0}{0.4} = \frac{-51.3°}{\text{sec}}$$

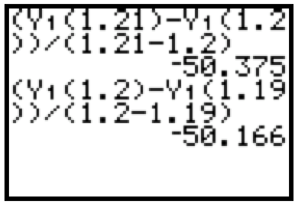
Each of these can be an approximation for the *instantaneous rate of change of elbow angle.* But they are all quite different. To get a better approximation, we use the elbow angle at t = 1.2 and a value **close to** t =1.2. To start, let's use t = 1.1 or t =1.3. Next, set your calculator to 3 decimal place accuracy and estimate the instantaneous rate of change more accurately. The calculus word for this instantaneous rate of change is called the **derivative.**

$$\text{Instantaneous rate of change } \approx \frac{d(1.3)-d(1.2)}{1.3-1.2} = \frac{85.2-90.3}{0.1} = \frac{-51°}{\text{sec}} \text{ or } \frac{d(1.2)-d(1.1)}{1.2-1.1} = \frac{95.3-90.3}{0.1} = \frac{-50°}{\text{sec}}$$

$$\text{Instantaneous rate of change } \approx \frac{d(1.21)-d(1.2)}{1.21-1.2} = \frac{-50.375°}{\text{sec}} \text{ or } \frac{d(1.2)-d(1.19)}{1.2-1.19} = \frac{-50.166°}{\text{sec}}$$

Note how our answers are getting closer together. To best estimate the *instantaneous rate of change of elbow angle* or the derivative of the elbow angle, let's learn how to do this easier on the TI-84 calculator.





It is important to be able to find the value of the function Y1 at different values of x. To accomplish this, use VARS | Y-VARS | Function | Y1 to paste in Y1 onto your screen. Then use parentheses and the value of x you want to evaluate.

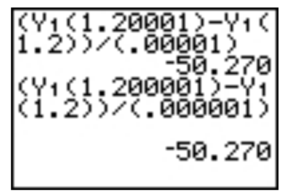
We can get the two answers closer together by "squeezing" into $t = 1.2$.

$$\text{Instantaneous rate of change} \approx \frac{d(1.201)-d(1.2)}{1.201-1.2} = \frac{-50.281°}{\text{sec}} \text{ or } \frac{d(1.2)-d(1.199)}{1.2-1.199} = \frac{-50.260°}{\text{sec}}$$

$$\text{Instantaneous rate of change} \approx \frac{d(1.2001)-d(1.2)}{1.2001-1.2} = \frac{-50.271°}{\text{sec}} \text{ or } \frac{d(1.2)-d(1.1999)}{1.2-1.1999} = \frac{-50.269°}{\text{sec}}$$

$$\text{Instantaneous rate of change} \approx \frac{d(1.20001)-d(1.2)}{1.20001-1.2} = \frac{-50.270°}{\text{sec}} \text{ or } \frac{d(1.2)-d(1.19999)}{1.2-1.19999} = \frac{-50.270°}{\text{sec}}$$

We have our best approximation when the two answers are the same. So the best approximation for the instantaneous rate of change of elbow angle (derivative) is: $\boxed{\frac{-50.270°}{\text{sec}}}$. Rather than do both of these calculations, decide on squeezing in from the left or the right. Do one of these calculations and then squeeze in closer and do another. If there is no change, you have your answer.



2. A boy is on a swing. Once the boy starts moving, the swing's distance d, measured in feet, off the ground depends on the number of seconds t from that time. The equation for d is $6-3\cos\frac{\pi}{3}t, t \geq 0$. We wish to know how fast the boy is moving towards or away from the ground at a given time t.

a. Find d when $t = 2$. Be sure you are in radian mode. $\boxed{\text{7.5 feet}}$

b. Estimate the instantaneous rate of change (derivative) of d at $t = 2$ by finding the average rate of change for
$t = 2$ to $t = 2.1$ $\qquad$ $t = 2$ to $t = 2.01$ $\qquad$ $t = 2$ to $t = 2.001$.

$$\boxed{\frac{d(2.1)-d(2)}{2.1-2} = \frac{2.634 \text{ ft}}{\text{sec}}} \qquad \boxed{\frac{d(2.01)-d(2)}{2.01-2} = \frac{2.712 \text{ ft}}{\text{sec}}} \qquad \boxed{\frac{d(2.001)-d(2)}{2.01-2} = \frac{2.720 \text{ ft}}{\text{sec}}}$$

c. Why can't the actual instantaneous rate of change of d with respect to t be calculated using this method?

$\boxed{\text{division by zero would occur}}$

d. Estimate the derivative of d with respect to t at $t = 4$ seconds accurate to 3 decimal places. Is the boy approaching the ground or moving away from it?

$$\boxed{\frac{d(4.00001)-d(4)}{.0001} = \frac{-2.721 \text{ ft}}{\text{sec}} \text{ moving toward the ground.}}$$

3. If you go into a glass specialty store, you will see that a pane of glass twice as long as another of the same type does not necessarily cost twice as much. Let x be the number of inches long of a 12 inch wide pane of glass and let C be the cost of the pane in dollars and cents. Assume that $C=\dfrac{0.2x^3-3.8x^2+30x}{150}$.

a. Find the price of 12 inch wide panes that are 2 ft long $\boxed{\$8.64}$, 4 ft long $\boxed{\$98.69}$, 6 ft long $\boxed{\$380.74}$

b. Find the average rate of change of the price in cost per foot for glass that changes from

3 ft to 3 ft, 1 inch	3 ft to 3 ft, 0.5 inch	3 ft to 3 ft, 0.1 inch
$\boxed{\dfrac{C(37)-C(36)}{1}=3.68}$	$\boxed{\dfrac{C\left(36\frac{1}{2}\right)-C(36)}{1/2}=3.620}$	$\boxed{\dfrac{C(36.1)-C(36)}{0.1}=3.572}$

c. As you continue this process, it seems that the average rate of change is approaching a number. What number is this and what is the name given to the rate of change?

$\boxed{\text{\$3.56 - instantaneous rate of change of cost per inch or the derivative of cost}}$

d. Estimate the derivative in price if x is 1 ft, 2 feet and x = 4 feet. Use proper units.

$\boxed{\text{1 ft}: \dfrac{C(12.0001)-C(12)}{.0001}=\dfrac{\$0.17}{\text{inch}}}$ $\boxed{\text{2 ft}: \dfrac{C(24.0001)-C(24)}{.0001}=\dfrac{\$1.29}{\text{inch}}}$ $\boxed{\text{4 ft}: \dfrac{C(48.0001)-C(48)}{.0001}=\dfrac{\$6.98}{\text{inch}}}$

e. Usually the more of a quantity you buy, the less you pay per unit. This is not true when purchasing glass. Give a reason why this is not true?

$\boxed{\text{Once glass gets too big, it is harder to make and store.}}$

4. Marge and Homer are exercising on stair-stepping machines. The total number of floors that Marge takes is modeled by the function: $M(t)=4.5\left(t+\sqrt{t}\right)$ where t is measured in minutes. Find the total number of floors that Marge has taken in 9 minutes and the derivative of M at t = 9 minutes. What are the units?

$$\boxed{M(9)=4.5\left(9+\sqrt{9}\right)=54 \text{ floors} \qquad \frac{M(9.0001)-M(9)}{.0001}=5.25 \text{ floors/min}}$$

Homer's workout is modeled by $H(t)=10-\dfrac{t^{3/2}}{10}$ where t is measured in minutes and $H(t)$ is measured in floors/min. Find the rate that Homer is working out at t = 9 minutes and the derivative of H at 9 minutes. What is this measuring?

$$H(9)=10-\frac{9^{3/2}}{10}=7.3 \text{ floors/min} \qquad \frac{H(9.0001)-H(9)}{.0001}=-0.45$$

The speed that Homer is working out is going down by .45 floors per minute per minute.

5. A tire is punctured by a nail. As the air leaks out, the distance d inches, between the rim of the tire and the road depends on the time t in minutes since the tire was punctured. Values of t and d are given in the following chart.

t min.	0	2	4	6	8	10	12	14	16
d in.	7.00	5.86	5.52	5.07	4.77	4.52	4.21	3.98	2.75

Approximately how fast is d changing when t = 10

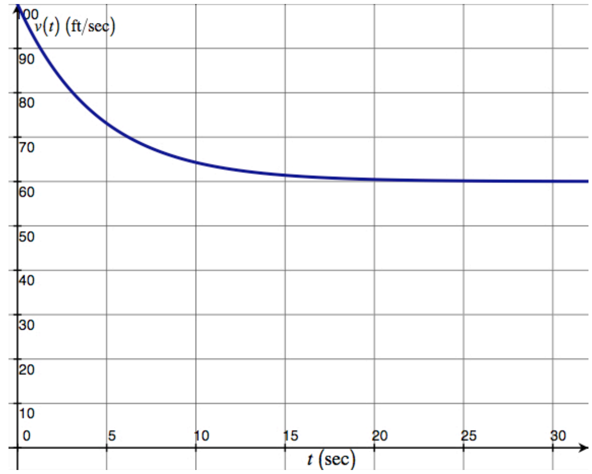
$$\boxed{\frac{4.52-4.77}{10-8}=\frac{-.125 \text{ in}}{\text{min}} \text{ or } \frac{4.21-4.52}{10-8}=\frac{-.155 \text{ in}}{\text{min}} \text{ or } \frac{4.21-4.77}{12-8}=\frac{-.14 \text{ in}}{\text{min}}}$$

t = 16

$$\boxed{\frac{2.75-3.98}{16-14}=\frac{-.615 \text{ in}}{\text{min}}}$$

Introductory Lesson 3

6. As you drive on a highway, you are traveling at 100 feet per second when you see a policeman. So you slow down to 60 feet/second. The diagram below shows the graph of your velocity $v(t)$ as a function of the number of seconds t since you started to slow down.



a. What does your velocity seem to be between $t = 20$ seconds and $t = 30$ seconds? $\boxed{60\text{ ft/sec}}$

b. About how far do you travel in the time interval [20, 30]? $\boxed{600\text{ ft}}$

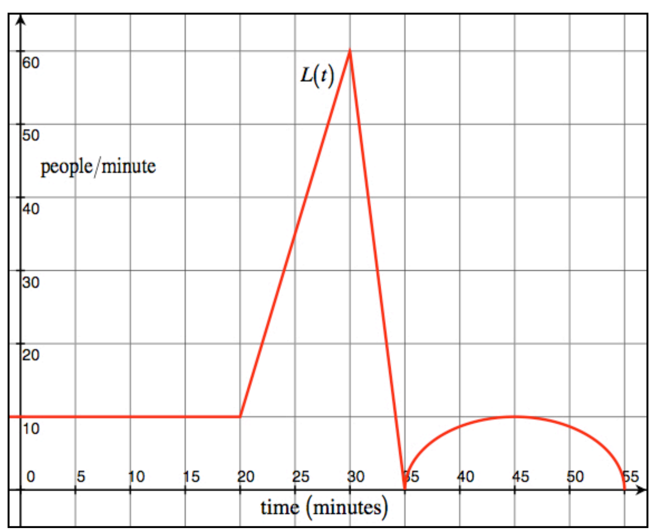
c. Explain why the answer above can be expressed as the area of a rectangular region of the graph? Shade the area. $\boxed{\text{base} = 20, \text{height} = 30 \quad 20(30) = 600}$

d. The distance you travel between $t = 0$ and $t = 30$ can be represented as the area of a region bounded by the graph. Count the number of rectangles in this region. Estimate the area of parts of rectangles to the nearest 0.1 square space. $\boxed{8.2 + 6.7 + 6.2 + 6 + 6 + 6 \approx 39.1 \text{ rectangles}}$

e. How many feet does each small rectangle on the graph represent? $\boxed{50 \text{ feet}}$. So about how far do you travel in the time [0, 30]? $\boxed{39.1(50) \approx 1{,}955 \text{ feet}}$

f. The problem above involves finding the *area under a curve*. This is calculated by estimating a product of a t-value and a y-value. Such a product is called a **definite integral of y with respect to t**. Since t is measured in seconds and y is measured in ft/second, the definite integral will be measured in $\boxed{\text{sec(feet/sec)} = \text{feet}}$.

7. People line up to purchase tickets at a concert. The graph to the right is $L(t)$, made of straight lines and a semicircle. $L(t)$ represents the rate that people join the line measured in people per minute. Calculate the definite integral of $L(t)$ with respect to t from the given values and explain what the number represents.



a. Definite integral of $L(t)$ from $t = 0$ to $t = 20$.
$\boxed{200 \text{ people line up from } t = 0 \text{ to } t = 20 \text{ minutes}}$

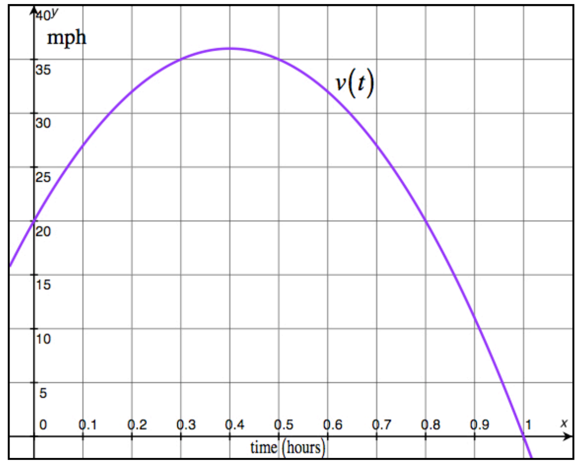
b. Definite integral of $L(t)$ from $t = 20$ to $t = 30$.
$\boxed{350 \text{ people line up from } t = 20 \text{ to } t = 30 \text{ minutes}}$

c. Definite integral of $L(t)$ from $t = 30$ to $t = 35$.
$\boxed{150 \text{ people line up from } t = 30 \text{ to } t = 35 \text{ minutes}}$

d. Definite integral of $L(t)$ from $t = 35$ to $t = 55$.
$\boxed{\approx 157 \text{ people line up from } t = 35 \text{ to } t = 55 \text{ minutes}}$

e. Definite integral of $L(t)$ from $t = 0$ to $t = 55$.
$\boxed{\approx 857 \text{ people line up from } t = 0 \text{ to } t = 55 \text{ minutes}}$

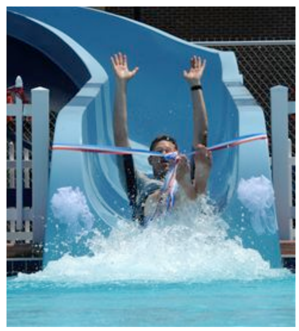
8. The figure below shows a graph of the velocity function $v(t) = -100t^2 + 80t + 20$ where t is in hours and $v(t)$ is in miles per hour. Estimate the definite integral of $v(t)$ with respect to t for time interval $t = 0$ hours to $t = 1$ hour. Be sure you specify units.

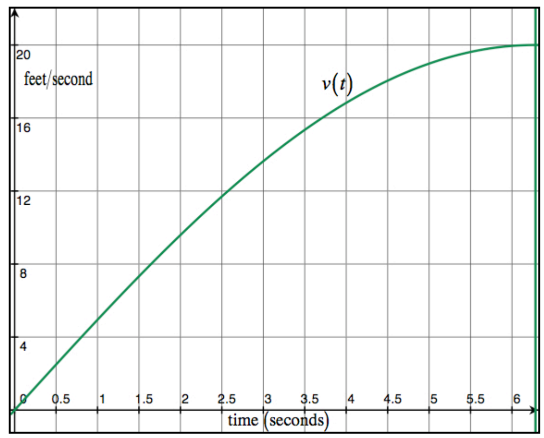


$$\boxed{\begin{array}{l} \text{Rectangles} \approx 4.9 + 6 + 6.7 + 7.2 + 7.2 + \\ \qquad 6.6 + 6 + 4.9 + 3.1 + 1.2 = 53.9 \\ 53.9(0.5) = 26.95 \text{ miles} \end{array}}$$

Let's put some concepts together.

9. James sits atop a sliding board which empties into a pool. At time $t = 0$ seconds, he pushes off. His velocity $v(t)$, measured in ft/sec is given by $v(t) = 20\sin(.25t)$. James hits the water at $t = 2\pi$ seconds.





a. How fast was he going when he hit the water? $\boxed{20 \text{ ft/sec}}$

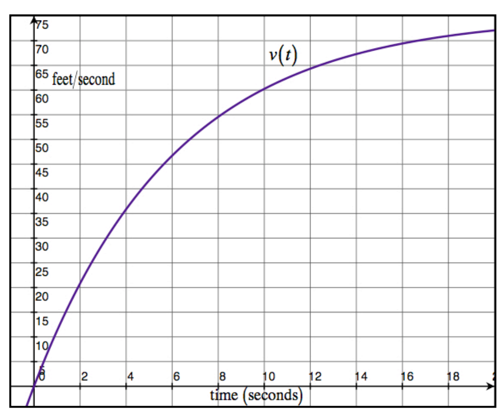
b. At approximately what rate was his speed changing at $t = \quad 2\pi$? $\boxed{\dfrac{v(2\pi + .0001) - v(2\pi)}{.0001} \approx 0}$

c. Explain the special name for answer b, and why this answer makes sense.

$\boxed{\text{Acceleration. As seen in the picture, when he hits the water, his speed is not changing.}}$

d. What is the approximate length of the sliding board? $\boxed{\begin{array}{l} \text{Approximately 39.8 rectangles.} \\ (39.8)(.5)(4) \approx 79.6 \text{ ft} \end{array}}$

10. A new boat when accelerated from a standing start travels with velocity $v(t) = 75(1 - 0.849^t)$ measured in ft/sec.



a. Draw the graph of the function in its domain [0, 20].

b. Approximately when will the boat reach 55 ft/sec? $\boxed{8 \text{ seconds}}$

c. About how far did the boat travel when it reaches 55 ft/sec?

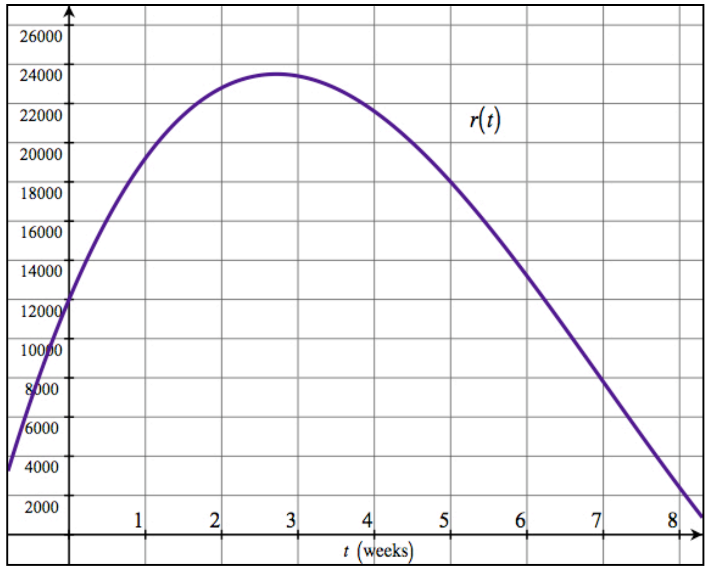
$\boxed{\approx 26.2 \text{ rectangles} \cdot (2)(5) = 262 \text{ feet}}$

d. At approximately what rate is the velocity changing when it reached 55 ft/sec? What is this called?

$\boxed{\dfrac{v(8.0001) - v(8)}{.0001} = 3.314 \text{ ft/sec}^2 \text{ - acceleration}}$

Introductory Lesson 4

11. Lady Tata comes out with a new song and puts it on YouTube. At time $t = 0$, the song is being downloaded at the rate of 12,000 times a week. The rate increases for a few weeks, but then starts to decrease. The rate of downloads $r(t)$ is shown by graph below.



a. What does the definite integral of $r(t)$ from 0 to 8 represent?

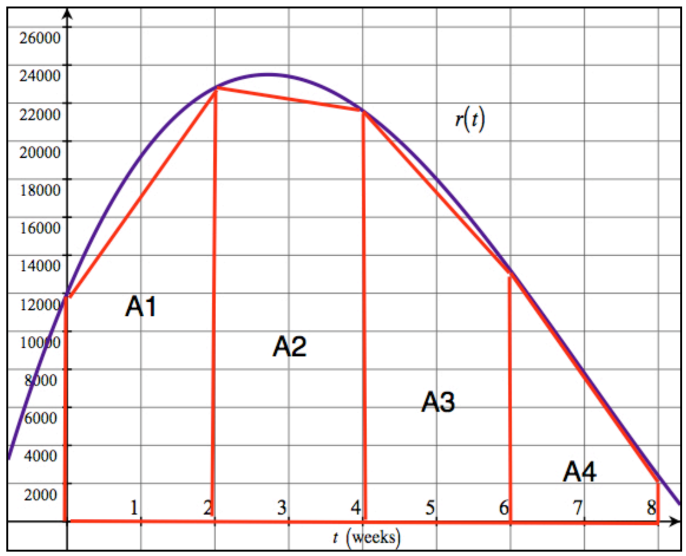
the total number of downloads of the song in the first 8 weeks.

b. Estimate this definite integral by counting and estimating rectangles.

≈ 67 rectangles
134,000 downloads

c. On the figure to the right, divide the region which represents the definite integral of $r(t)$ from 0 to 8 into 4 trapezoids of equal width.

d. Now estimate the value of the definite integral of $r(t)$ from 0 to 8 by estimating the areas of the 4 trapezoids.Is this an under or over-estimation of the definite integral?



$A1 \approx .5(2)(12000+23000) = 35000$
$A2 \approx .5(2)(23000+21800) = 44800$
$A3 \approx .5(2)(21800+13000) = 34800$
$A4 \approx .5(2)(13000+2000) = 15000$

$A \approx 129{,}600$ (underestimation)

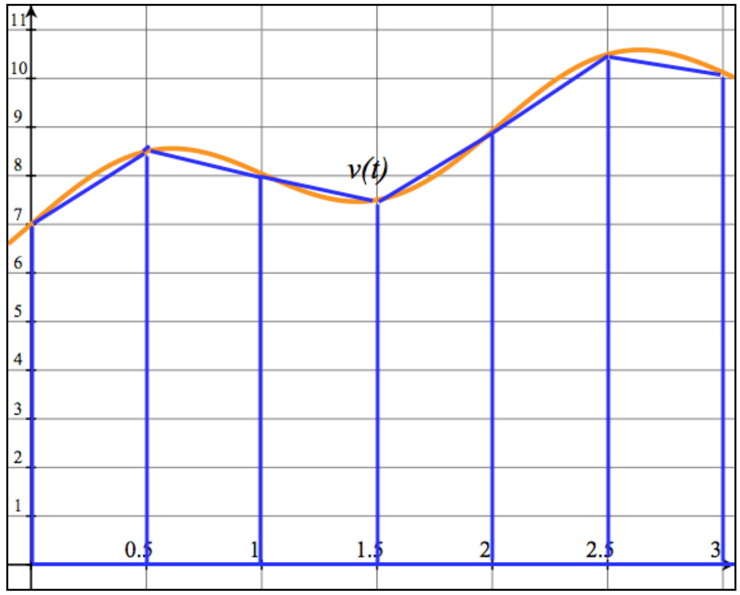
e. Rather than estimating, here are the values of $r(t)$ at each integer value of t. Estimate the definite integral of $r(t)$ from 0 to 8 by finding the areas of the 8 trapezoids. Is answer d) or e) a better estimation?

t	0	1	2	3	4	5	6	7	8
$r(t)$	12,000	19,200	22,800	23,400	21,600	18,000	13,200	7,800	2,400

$A1 = .5(1)(12000+19200) = 15600$	$A5 = .5(1)(21600+18000) = 19800$	
$A2 = .5(1)(19200+22800) = 21000$	$A6 = .5(1)(18000+13200) = 15600$	
$A3 = .5(1)(22800+23400) = 23100$	$A7 = .5(1)(13200+7800) = 10500$	
$A4 = .5(1)(23400+21600) = 22500$	$A8 = .5(1)(7800+2400) = 5100$	$A \approx 133{,}200$ (better answer)

12. Izzy Fast enters a marathon (26.2 miles). He starts out slowly at 7 miles per hour, and speeds up or slows down based on how he feels. His speed can be approximated by $v(t) = t + 7 + \sin(3.1t)$ where t is the number of hours from the start of the race and $v(t)$ is in miles per hour.

a. Sketch the graph of $v(t)$ for three hours (the amount of time he actually runs).



b. What does the definite integral from 0 to 3 of $v(t)$ represent?

the distance Izzy runs in 3 hours.

c. On your diagram, draw in 6 equally spaced trapezoids to represent this definite integral. Will using these trapezoids under or over-Estimate the definite integral?

underestimate

d. Estimate the value of the definite integral by calculating the areas of the 6 trapezoids (one decimal place).

$\text{A}1 = (.5)(.5)(7 + 8.5) = 3.9$ $\quad \text{A}3 = (.5)(.5)(8. + 7.5) = 3.9$ $\quad \text{A}5 = (.5)(.5)(8.9 + 10.5) = 4.9$

$\text{A}2 = (.5)(.5)(8.5 + 8) = 4.1$ $\quad \text{A}4 = (.5)(.5)(7.5 + 8.9) = 4.1$ $\quad \text{A}6 = (.5)(.5)(10.5 + 10.1) = 5.2$ $\quad A \approx 26.1$ miles

e. Show that this calculation can be made easier by factoring:

$A \approx .5(.5)[7 + 2(8.5) + 2(8) + 2(7.5) + 2(8.9) + 2(10.5) + 10.1] = 26.0$ miles (roundoff error explains the difference)

So the trapezoidal rule is as follows: The definite integral from $x = a$ to $x = b$ is approximately equal to $T_n = \frac{1}{2}(b)\left[f(a) + 2f(x_1) + 2f(x_2) + 2f(x_3) + ... + 2f(x_{n-1}) + f(b)\right]$ where n is the number of rectangles and b is the base. Verbally, double each value of $f(x)$ except the first and the last. Add these numbers and multiply by one-half the base.

13. You are taking a long trip in a car. Your odometer doesn't work but your speedometer does. Every 15 minutes, you record your speed.

time	3:00	3:15	3:30	3:45	4:00	4:15	4:30	4:45
mph	38	35	55	22	15	40	40	26

a. Estimate the distance you travel between 3:00 p.m. and 4:45 p.m. Use the trapezoid rule.

$$D \approx \frac{1}{2}\left(\frac{1}{4}\right)(38 + 2(35) + 2(55) + 2(22) + 2(15) + 2(40) + 2(40) + 26) = 59.75 \text{ miles}$$

b. Why is this estimate a poor one? What would you have to do to make it more accurate?

Too much time passes between recording of speeds. To get more accuracy, you need more speeds so the time between the any 2 recordings are smaller.

14. When a plane lands on an aircraft carrier, it must stop very quickly. A pilot lands on a carrier with the following speeds s in feet/second.

t sec	0	0.5	1.0	1.5	2.0	2.5	3
s ft/sec	420	330	230	175	110	55	0

a. Estimate the instantaneous change of velocity (derivative) of the plane's speed at $t = 1$ second. Show three different answers. What special word in physics represents this change of velocity?

$$\frac{s(1)-s(.5)}{1-.5}=\frac{230-330}{.5}=\frac{-200\text{ ft}}{\sec^2} \text{ or } \frac{s(1.5)-s(1)}{1.5-1}=\frac{175-230}{.5}=\frac{-110\text{ ft}}{\sec^2}$$

$$\text{or } \frac{s(1.5)-s(.5)}{1.5-.5}=\frac{175-330}{1}=\frac{-155\text{ ft}}{\sec^2}$$

deceleration

b. Estimate how far the plane travels as it comes to a stop. Is there any danger of it running off the end of the 600 foot carrier flight deck? You should be using the trapezoidal rule.

$$d \approx (.5)(.5)\big(420+2(330)+2(230)+2(175)+2(110)+2(55)+0\big)=555 \text{ feet}$$

Assuming that the speed decreases quite quickly, there should be enough room

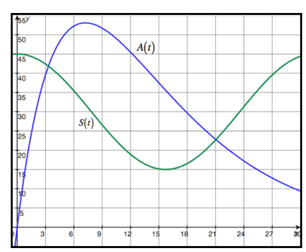
15. The amount of water that flows over the spillway of a dam is difficult to measure. However, if we know the flow rate and length of time the water has been flowing, we can determine this amount. Suppose the flow rate (measured in thousands of cubic feet per hour)is recorded every 2 hours for a 24-hour period. Use the trapezoidal rule to estimate the definite integral of the flow rate, being sure to specify proper units.

time	12 am	2 am	4 am	6 am	8 am	10 am	12 pm	2 pm	4 pm	6 pm	8 pm	10 pm	12 am
gallons/hr	6	8	13	16	14	12	9	5	11	13	14	9	6

$$\frac{1}{2}(2)\big[6+2(8)+2(13)+2(16)+2(14)+2(12)+2(9)+2(5)+2(11)+2(13)+2(14)+2(9)+6\big]$$

260,000 gallons

16. A new car called the Sexus has its plant next to its only dealership. Completed cars are moved immediately to the lot to be sold. Suppose at the start of the month of May there are 100 cars on the lot waiting to be sold. Cars come off the assembly line at the rate of $A(t)=20t\left(2^{-0.2t}\right)$ and cars are sold at the rate of $S(t)=30+15\cos(0.2t)$ where t represents the day of the month (for May 1, $t = 0$ and for May 31, $t = 30$). Use the trapezoidal rule (with trapezoid width = 3) to estimate the number of cars on the lot available to be sold on May 22.



$$\text{Cars on lot}: 100+\frac{1}{2}(3)\big[A(0)+2A(3)+2A(6)+2A(9)+2A(12)+2A(15)+2A(18)+A(21)\big]\approx 903$$

$$\text{Cars sold}: \frac{1}{2}(3)\big[S(0)+2S(3)+2S(6)+2S(9)+2S(12)+2S(15)+2S(18)+S(21)\big]\approx 566$$

Cars available to be sold $\approx 903-566=337$