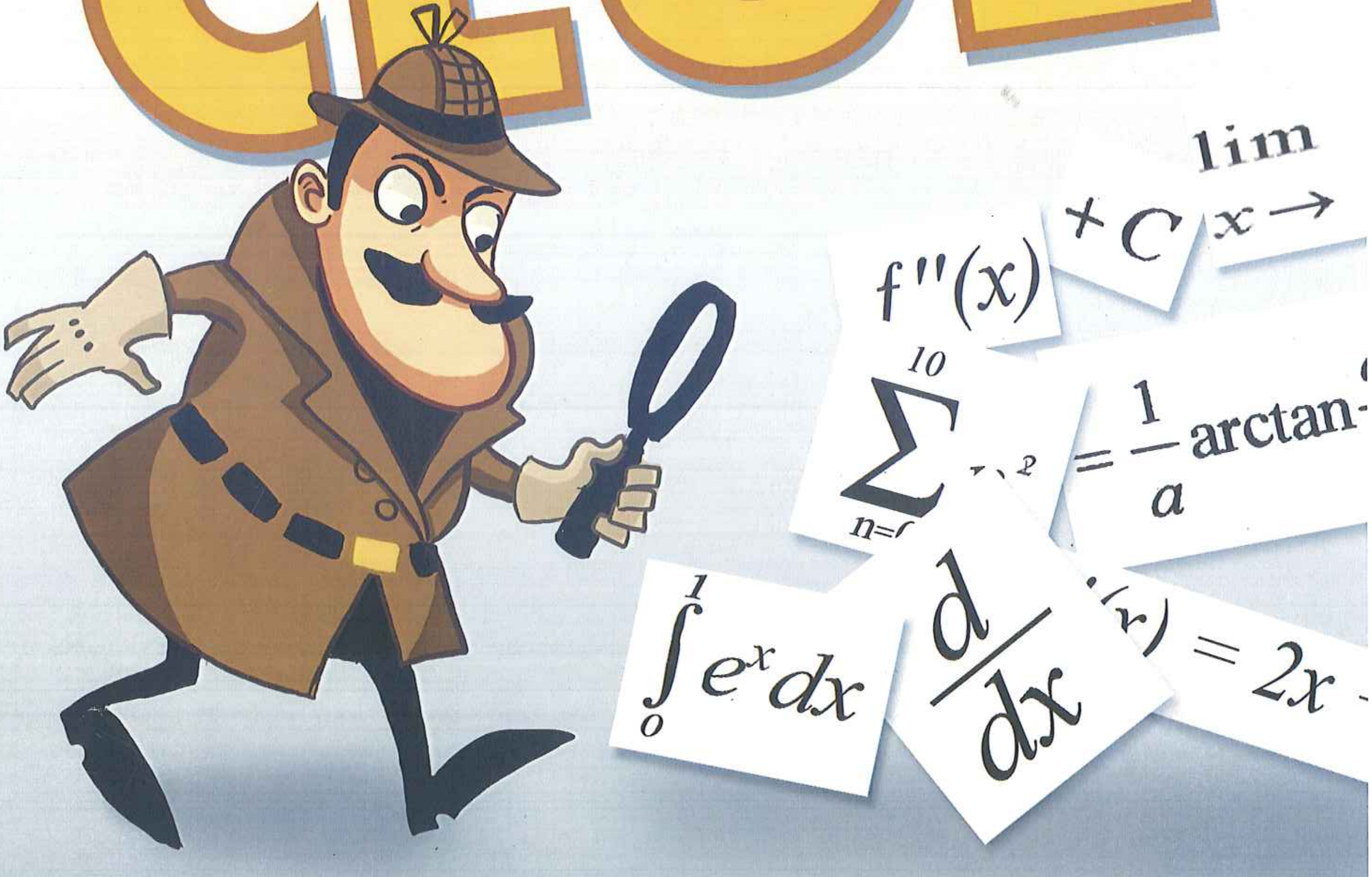
A.P. CALCULUS
CLUE



Preparation Problems for the Advanced Placement™ AB & BC Calculus Exams - *with a twist!*

by Stu Schwartz
sschwartz8128@verizon.net

AB Clue Problem Set # 1 Solutions

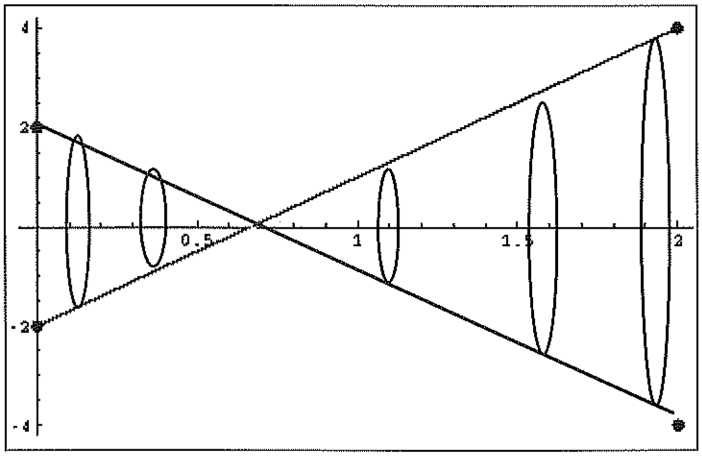
Suspect Problem: Find the derivative of $f(x)=2\ln x^4+4x$ at $x=0.35$. Round to the nearest integer.

$$
\begin{aligned}
&f'(x)=2\left(\frac{1}{x^4}\right)\left(4x^3\right)+4\\
&f'(x)=\frac{8}{x}+4\\
&f'(.35)=26.857\approx 27
\end{aligned}
$$

The answer is 27.

Location Problem: Find the volume if the region enclosing $y=|3x-2|, x=0$, and $x=2$ is rotated about the x-axis. Round to the nearest integer.

$$
\begin{aligned}
&V=\pi\int_0^2(3x-2)^2dx\\
&V=\pi\int_0^2\left(9x^2-12x+4x\right)dx=3x^3-6x^2+4x\Big]_0^2\\
&V=8\pi=25.133\approx 25
\end{aligned}
$$



The answer is 25.

Treasure Problem: Find $\int_0^4\frac{15}{\sqrt{2x+1}}dx$

$$
\begin{aligned}
&\int_0^4\frac{15}{\sqrt{2x+1}}dx\\
&15\int_0^4(2x+1)^{-1/2}dx && u=2x+1, du=2dx\\
& && x=4\Rightarrow u=9\\
&\frac{15}{2}\int_1^9 u^{-1/2}du=15u^{1/2}\Big]_1^9 && x=0\Rightarrow u=0\\
&15(3-1)=30
\end{aligned}
$$

The answer is 30.

AB Clue Problem Set # 1

Suspect Problem: Find the derivative of $f(x)=2\ln(x^4)+4x$ at $x=0.35$. Round to the nearest integer.

$f'(x) = 2 \cdot \frac{1}{x^4} \cdot 4x^3 + 4$

$f'(0.35) = \frac{2}{0.35^4} \cdot 4(0.35)^3 + 4 = 27$

The answer is: 27. Cross out that suspect number on your clue card and write # 1 as your set.

Location Problem: Find the volume if the region enclosing $y=|3x-2|$, $x=0$, and $x=2$ is rotated about the x-axis. Round to the nearest integer.

$u = 3x-2$

$du = 3\,dx$

$u(0) = -2$

$u(2) = 4$

$3x-2=0$

$x = \frac{2}{3}$

$\pi\int_0^2 (3x-2)^2\,dx = \frac{\pi}{3}\int_{-2}^{4} u^2\,du$

$= 8\pi \approx 25$

$= \frac{\pi}{3}\left(\frac{u^3}{3}\Big|_{-2}^{4}\right)$

$= \left[\frac{4^3}{3} - \frac{(-2)^3}{3}\right]\frac{\pi}{3}$

$= \left[\frac{64}{3} + \frac{8}{3}\right]\frac{\pi}{3} = \left(\frac{72}{3}\right)\frac{\pi}{3} = \frac{72}{9}\pi = 8\pi \approx 25$

The answer is: 25. Cross out that location number on your clue card and write # 1 as your set.

Treasure Problem: Find $\int_0^4 \frac{15}{\sqrt{2x+1}}dx$

$u = 2x+1$

$du = 2\,dx$

$u(0) = 1$

$u(4) = 9$

$= \frac{15}{2}\int_0^4 2(2x+1)^{-1/2}\,dx$

$= \frac{15}{2}\int_1^9 u^{-1/2}\,du = \frac{15}{\cancel{2}}\left(\cancel{2}u^{1/2}\right)\Big|_1^9$

$= 15(\sqrt{9} - \sqrt{1}) = 15(3-1)$

$= 15 \cdot 2 = 30$

The answer is: 30. Cross out that treasure number on your clue card and write # 1 as your set.

Stu Schwartz

AB Clue Problem Set # 2

Calc 1 #3

Suspect Problem: The acceleration of an object is given by $a(t) = 6\sin t$ with initial velocity of -9.5. Find the distance the object travels on the interval $[0,\pi]$ to the nearest integer.

$t=0 \quad v(t) = -9.5$

need calculator

$a(t) = 6\sin(t)$

$v(t) = -6\cos t + C$

$-9.5 = -6\cos 0 + C$

$-9.5 = -6(1) + C$

$-3.5 = C$

$-6\cos t - 3.5 = 0$

$s(t) = \int_0^{\pi} |-6\cos t - 3.5|\,dt$

$= \int_0^{2.194} |-6\cos t - 3.5|\,dt + \int_{2.194}^{\pi} -6\cos t - 3.5\,dt$

$= \int_{2.194}^{0} -6\cos t - 3.5\,dt + \int_{2.194}^{\pi} -6\cos t - 3.5\,dt$

$= 12.551 + 1.556 \approx 14$

The answer is: 14. Cross out that suspect number on your clue card and write # 2 as your set.

Location Problem: Find the slope of the line normal to $y^2 + 2x = 2y + x^3 y + 440$ at $x = 0$ when $y > 0$.

$2y\frac{dy}{dx} + 2 = 2\frac{dy}{dx} + x^3\frac{dy}{dx} + 3x^2 y$

$(2y - 2 - x^3)\frac{dy}{dx} = 3x^2 y - 2$

$\frac{dy}{dx} = \frac{3x^2 y - 2}{2y - 2 - x^3}$

@$(0, 22)$ $\frac{dy}{dx} = \frac{3(0^2)22 - 2}{2(22) - 2 - 0^3} = \frac{-2}{42} = \frac{-1}{21}$ $\perp m = 21$

pt when $x = 0$

$y^2 + 2(0) = 2y + 0^3 y + 440$

$y^2 = 2y + 440$

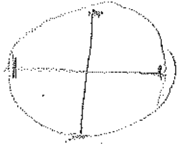
$y^2 - 2y - 440 = 0$

$(y+20)(y-22) = 0$

$y = -20 \quad \boxed{y = 22}$

The answer is: 21. Cross out that location number on your clue card and write # 2 as your set.

Treasure Problem: An aquarium is built with a 1.58 foot radius circle as a base with center at the origin. The aquarium is built with equilateral triangles as cross sections perpendicular to the *x*-axis. Find the volume of the aquarium to the nearest foot.



$x^2 + y^2 = 1.58^2$

$y = \sqrt{1.58^2 - x^2}$

base $= \sqrt{1.58^2 - x^2} - (-\sqrt{1.58^2 - x^2}$

$= 2\sqrt{1.58^2 - x^2}$

$A(x) = \frac{1}{2} b h$

$= \frac{1}{\cancel{2}}(\cancel{2}\sqrt{1.58^2 - x^2})(\sqrt{1.58^2 - x^2}\sqrt{3})$

$\int_{-1.58}^{1.58} (1.58^2 - x^2)\sqrt{3}\,dx \approx 9.109$

The answer is: 9. Cross out that location number on your clue card and write # 2 as your set.

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Stu Schwartz

AB Clue Problem Set # 2 Solutions

Suspect Problem: The acceleration of an object is given by $a(t)=6\sin t$ with initial velocity of -9.5. Find the distance the object travels on the interval $[0,\pi]$ to the nearest integer.

$$v(t)=\int 6\sin t\,dt=-6\cos t+C \qquad s(t)=\int_0^{\pi}\left|-6\cos t-3.5\right|dt$$

$$v(0)=-6+C=-9.5\Rightarrow C=-3.5 \qquad s(t)=\int_0^{2.194}(6\cos t+3.5)\,dt+\int_{2.194}^{\pi}(-6\cos t-3.5)\,dt$$

$$v(t)=-6\cos t-3.5 \qquad s(t)=6\sin t+3.5t\Big]_0^{2.194}-\Big[6\sin t+3.5t\Big]_{2.194}^{\pi}=14.106\approx 14$$

The answer is 14.

Location Problem: Find the slope of the line normal to $y^2+2x=2y+x^3y+440$ at $x=0$ when $y>0$.

$$2y\frac{dy}{dx}+2=2\frac{dy}{dx}+x^3\frac{dy}{dx}+3x^2y$$

$$\frac{dy}{dx}\left(2y-x^3-2\right)=3x^2y-2\Rightarrow\frac{dy}{dx}=\frac{3x^2y-2}{2y-x^3-2}$$

$$\frac{dy}{dx}_{(x=0)}=\frac{-2}{42}=\frac{-1}{21}\Rightarrow \text{slope of normal line is 21}$$

$$y^2+2x=2y+x^3y+440$$

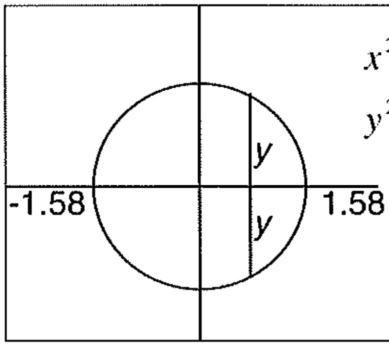
$$y^2=2y+440$$

$$y-2y-440=0$$

$$(y-22)(y+20)=0\Rightarrow y=22$$

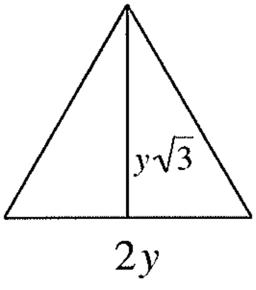
The answer is 21.

Treasure Problem: An aquarium is built with a 1.58 foot radius circle as a base with center at the origin. The aquarium is built with equilateral triangles as cross sections perpendicular to the *x*-axis. Find the volume of the aquarium to the nearest foot.



$$x^2+y^2=1.58^2$$

$$y^2=1.58^2-x^2$$



$$A=\frac{1}{2}(2y)\left(y\sqrt{3}\right)$$

$$V=\sqrt{3}\int_{-1.58}^{1.58}\left(1.58^2-x^2\right)dx=9.106\text{ ft}^3$$

The answer is 9.

AB Clue Problem Set # 3 Solutions

Suspect Problem: If $F(x)=\int\frac{-12\sin x}{\cos^2 x}\,dx+C$ and $F(0)=-4$, find C.

$$F(x)=\int\frac{-12\sin x}{\cos^2 x}\,dx=-12\int\left(\frac{\sin x}{\cos x}\right)\left(\frac{1}{\cos x}\right)dx$$
$$F(x)=-12\int\tan x\sec x\,dx=-12\sec x+C$$
$$F(0)=-12(1)+C=-4\Rightarrow C=8$$

The answer is 8.

Location Problem: Find $\lim\limits_{x\to3}\frac{2x^3-5x^2+7}{3\sin\left(\frac{\pi x}{6}\right)-\cos(\pi x)}$

$$\lim_{x\to3}\frac{2x^3-5x^2+7}{3\sin\left(\frac{\pi x}{6}\right)-\cos(\pi x)}=\frac{2(27)-5(9)+7}{3\sin\left(\frac{\pi}{2}\right)-\cos 3\pi}=\frac{54-45+7}{3+1}=4$$

The answer is 4.

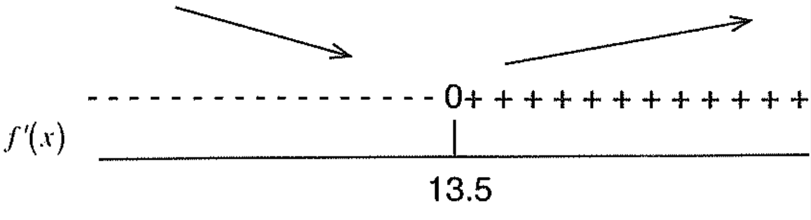
Treasure Problem: Given $f(x)=x^2-27x-28$, find the largest integer in which f is decreasing.

$f'(x)=2x-27$

$2x-27=0$

$2(x-13.5)=0$

$x=13.5$



The answer is 13.

AB Clue Problem Set # 3

Suspect Problem: If $F(x)=\int\frac{-12\sin x}{\cos^2 x}\,dx+C$ and $F(0)=-4$, find C.

$u = \cos x$

$du = -\sin x\, dx$

$F(x) = \int \frac{-12\sin x}{\cos^2 x}\,dx + C$

$F(x) = 12\int u^{-2}\,du + C$

$= 12\left(\frac{u^{-1}}{-1}\right) + C$

$= \frac{-12}{u} + C = \frac{-12}{\cos x} + C$

$F(0) = -4$

$-4 = \frac{-12}{\cos 0} + C$

$-4 = \frac{-12}{(1)} + C$

$\boxed{8 = C}$

The answer is: 8. Cross out that suspect number on your clue card and write # 3 as your set.

Location Problem: Find $\lim\limits_{x\to 3}\frac{2x^3-5x^2+7}{3\sin\left(\frac{\pi x}{6}\right)-\cos(\pi x)}$ $= \frac{2(3^3)-5(3^2)+7}{3\sin\left(\frac{3\pi}{6}\right)-\cos(3\pi)}$

$= \frac{2(27)-45+7}{3(1)-(-1)} = \frac{54-45+7}{3+1} = \frac{16}{4} = \boxed{4}$

The answer is: 4. Cross out that location number on your clue card and write # 3 as your set.

Treasure Problem: Given $f(x)=x^2-27x-28$, find the largest integer in which f is decreasing.

$f'(x) = 2x - 27$

$0 = 2x - 27$

$\frac{27}{2} = x$

$13.5 = x$

$f'(x)$ ↘ ↗

$f'(0) = -$ | 13.5 | $f'(15) = +$

f(x) is decreasing to $x = 13.5$

Largest integer when f is decreasing

is $\boxed{x = 13}$

The answer is: 13. Cross out that treasure number on your clue card and write # 3 as your set.

Stu Schwartz

AB Clue Problem Set # 4 Solutions

Suspect Problem: Find the value of a that makes the function continuous.

$$
\begin{aligned}
&f(x)=\begin{cases}\ln x+a, x>e\\ \dfrac{2x}{e}, x\le e\end{cases}\\
&\lim_{x\to e^+} f(x)=\lim_{x\to e^-} f(x)\\
&\ln e+a=\frac{2e}{e}\\
&1+a=2\\
&a=1
\end{aligned}
$$

The answer is 1.

Location Problem: Find the derivative of $\int_x^{\pi/4}(3t+2)\,dt$ at $x=-\frac{20}{3}$.

$$
\begin{aligned}
&\frac{d}{dx}\left(\int_x^{\pi/4}(3t+2)\,dt\right)=\frac{d}{dx}\left(\int_{\pi/4}^{x}(-3t-2)\,dt\right)=-3x-2\\
&-3\left(\frac{-20}{3}\right)-2=20-2=18
\end{aligned}
$$

The answer is 18.

Treasure Problem: The velocity of a particle is given by $v(t)=14e^{-t}+t$. Find the total distance traveled by the particle from $t=1$ to $t=5$ to the nearest integer.

Since e^{-t} is always positive, the particle is always traveling in the positive direction so displacement = distance.

$$
\begin{aligned}
&D=\int_1^5\left(14e^{-t}+t\right)dt=-14e^{-t}+\frac{t^2}{2}\Bigg]_1^5\\
&D=\frac{-14}{e^5}+\frac{25}{2}+\frac{14}{e}-\frac{1}{2}=17.056\approx 17
\end{aligned}
$$

The answer is 17.

AB Clue Problem Set # 4

Suspect Problem: Find the value of a that makes the function continuous.

$$f(x)=\begin{cases}\ln x+a, x>e\\ \frac{2x}{e}, x\le e\end{cases}$$

$\ln e + a = \frac{2e}{e}$

$1+a = 2$

$a = 2-1 = 1$

The answer is: 1. Cross out that suspect number on your clue card and write # 4 as your set.

Location Problem: Find the derivative of $\int_x^{\pi/4}(3t+2)\,dt$ at $x=-\frac{20}{3}$.

$\frac{d}{dx}\int_x^{\pi/4}(3t+2)dt = \frac{d}{dx}\left(-\int_{\pi/4}^{x}(3t+2)dt\right)$

$= -(3x+2) = -3x-2$

$= +\cancel{3}\left(+\frac{20}{\cancel{3}}\right)-2$

$= 20-2 = 18$

The answer is: 18. Cross out that location number on your clue card and write # 4 as your set.

Treasure Problem: The velocity of a particle is given by $v(t)=14e^{-t}+t$. Find the total distance traveled by the particle from $t=1$ to $t=5$ to the nearest integer.

$\int_1^5 |14e^{-t}+t|\,dt =$

e to any power positive
so $14e^{-t}+t$ always positive

$-14e^{-t}+\frac{t^2}{2}\Big|_1^5 = \left(-14e^{-5}+\frac{5^2}{2}\right)-\left(-14e^{-1}+\frac{(1)^2}{2}\right)$

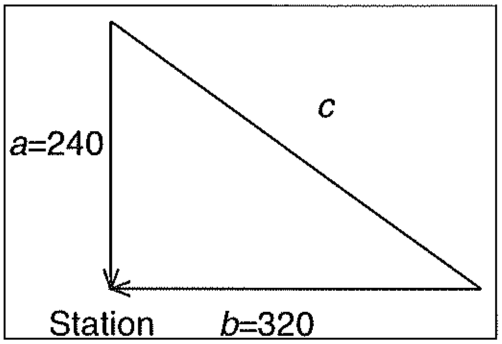
$= \frac{-14}{e^5}+\frac{25}{2}+\frac{14}{e}-\frac{1}{2} = \frac{-14}{e^5}+\frac{14}{e}+12 = 17.656$

The answer is: 17. Cross out that treasure number on your clue card and write # 4 as your set.

Stu Schwartz

AB Clue Problem Set # 5 Solutions

Suspect Problem: Two trains are traveling at approximately 164 mph towards a station. Train A is traveling south and is 240 miles from the station while train B is traveling west and is 320 miles from the station. To the nearest integer, how fast is the distance between the two trains changing at this time? *Reduce your answer by a factor of 10.*



$$c = a^2 + b^2$$

$$2c\frac{dc}{dt} = 2a\frac{da}{dt} + 2b\frac{db}{dt}$$

$$40\frac{dc}{dt} = 240(-164) + 320(-164)$$

$$40\frac{dc}{dt} = -91840 \Rightarrow \frac{dc}{dt} = -229.6 \approx -230$$

$$c = a^2 + b^2$$

$$c = 57600+102400$$

$$c = 400$$

The answer is 23.

Location Problem: Find the smallest positive integer in the domain of $f(x) = \dfrac{\sin^2 x}{\sqrt{x^2 - 28x - 29}}$.

$$x^2 - 28x - 29 > 0$$

$$(x-29)(x+1) > 0$$

$$x > 29$$

The answer is 30.

Treasure Problem: Find to the nearest integer the value of C that satisfies $(y-6)\,dy = \dfrac{(x+4)}{y-3}dx,\ y\left(\dfrac{-1}{12}\right) = 2$

$$(y^2 - 9y + 18)dy = (x+4)dx$$

$$\frac{y^3}{3} - \frac{9y^2}{2} + 18y = \frac{x^2}{2} + 4x + C$$

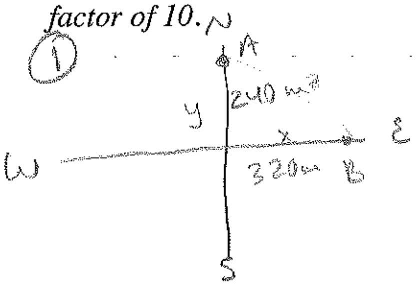
$$\frac{8}{3} - \frac{36}{2} + 36 = \frac{\frac{1}{144}}{2} + 4\left(\frac{-1}{12}\right) + C$$

$$C = 20.997 \approx 21$$

The answer is 21.

AB Clue Problem Set # 5

Suspect Problem: Two trains are traveling at approximately 164 mph towards a station. Train A is traveling south and is 240 miles from the station while train B is traveling west and is 320 miles from the station. To the nearest integer, how fast is the distance between the two trains changing at this time? *Reduce your answer by a factor of 10.*



② $x^2 + y^2 = z^2$

$\cancel{2}x\frac{dx}{dt} + \cancel{2}y\frac{dy}{dt} = \cancel{2}z\frac{dz}{dt}$

$320(-164) + 240(-164) = 400\frac{dz}{dt}$

$-91840 = 400\frac{dz}{dt}$

$-229.6 = \frac{dz}{dt}$

③ moment in time

$\frac{dx}{dt} = \frac{dy}{dt} = 164$

$x = 320$

$y = 240$

$320^2 + 240^2 = z^2$

$400 = z$

④ $\boxed{\text{ans} = \frac{230}{10} = 23}$

The answer is: 23. Cross out that suspect number on your clue card and write # 5 as your set.

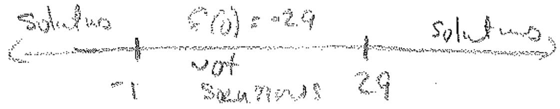
Location Problem: Find the smallest positive integer in the domain of $f(x) = \dfrac{\sin^2 x}{\sqrt{x^2 - 28x - 29}}$.

$x^2 - 28x - 29 > 0$

$(x-29)(x+1) > 0$

$x - 29 > 0 \quad x + 1 > 0$

$\boxed{x > 29} \quad x > -1$



$x = 30$

The answer is: 30. Cross out that location number on your clue card and write # 5 as your set.

Treasure Problem: Find to the nearest integer the value of C that satisfies $(y-6)\,dy = \dfrac{(x+4)}{y-3}dx,\ y\left(\dfrac{-1}{12}\right) = 2$.

$(y-6)\,dy = \frac{(x+4)}{(y-3)}dx$

$(y-3)(y-6)\,dy = x+4\,dx$

$\int(y^2 - 9y + 18)\,dy = \int(x+4)\,dx$

$\frac{y^3}{3} - \frac{9y^2}{2} + 18y = \frac{x^2}{2} + 4x + C$

$\frac{2^3}{3} - \frac{9(2^2)}{2} + 18(2) = \left(\frac{-1}{12}\right)\frac{1}{2} + 4\left(\frac{-1}{12}\right) + C$ X Y

$\frac{8}{3} - 18 + 36 = +\frac{1}{24} - \frac{4}{12} + C$

$\frac{64}{24} + 18 = +\frac{1}{24} - \frac{8}{24} + C$

$\frac{64}{24} + 18 = \frac{-2}{24} + C$

$\frac{71}{24} + 18 = C$

$C \approx 20.958$

$C \approx 21$

The answer is: 21. Cross out that treasure number on your clue card and write # 5 as your set.

Stu Schwartz

AB Clue Problem Set # 6 Solutions

Suspect Problem: Find $\boxed{\lim_{x\to 4}\left(\frac{2x^3-9x^2+x+12}{x-4}\right)}$

$$\lim_{x\to 4}\left(2x^2-x-3\right)=25 \qquad \begin{array}{r|rrrr} 4 & 2 & -9 & 1 & 12 \\ & & 8 & -4 & -12 \\ \hline & 2 & -1 & -3 & 0 \end{array}$$

The answer is 25.

Location Problem: Using the trapezoid method, approximate the area under $f(x)$ on [0,8] to the nearest integer given the following:

x	0	1	2	3	5	7	8
$f(x)$	3.4	2.7	6.2	5.3	1.3	2.1	4.8

$$A=\frac{1}{2}(3.4+2.7)+\frac{1}{2}(2.7+6.2)+\frac{1}{2}(6.2+5.3)+\frac{1}{2}(2)(5.3+1.3)+\frac{1}{2}(2)(1.3+2.1)+\frac{1}{2}(2.1+4.8)$$
$$A=26.7\approx 27$$

The answer is 27.

Treasure Problem: The drop in blood pressure of a typical patient who is given a certain medication is given by $D(x)=.028x^2(19-x)$ where x is the amount of medication in cubic centimeters. What is the maximum drop in blood pressure for this patient to the nearest integer?

$D(x) = .532x^2 - .028x^3$

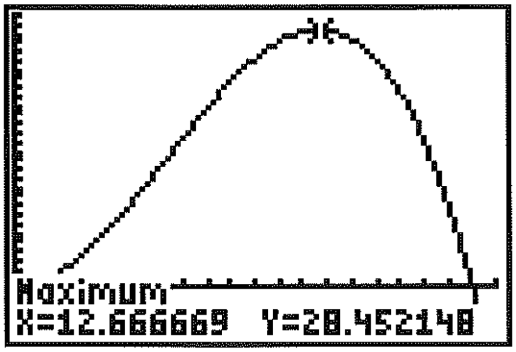
$D'(x) = 1.064x - .084x^2 = 0$

$D'(x) = x(1.064 - .084x) = 0$

$x = 0, x = 12\tfrac{2}{3}$

D is increasing on $\left[0,12\tfrac{2}{3}\right)$, decreasing on $\left(12\tfrac{2}{3},\infty\right)$

Maximum blood pressure drop at $D\left(12\tfrac{2}{3}\right)\approx 28$



The answer is 28.

Stu Schwartz

AB Clue Problem Set # 6

Suspect Problem: Find $\lim\limits_{x\to 4}\left(\dfrac{2x^3-9x^2+x+12}{x-4}\right)$

$$2x^2-1x-3$$
$$x-4\,\overline{)\,2x^3-9x^2+x+12}$$
$$-(2x^3-8x^2)$$
$$-x^2+x$$
$$-(-x^2+4x)$$
$$-3x+12$$
$$-3x+12$$
$$0$$

$$=\lim_{x\to 4}\left(2x^2-x-3\right)$$
$$=2(4^2)-4-3$$
$$=2(16)-4-3=32-7=25$$

The answer is: 25. Cross out that suspect number on your clue card and write # 6 as your set.

Location Problem: Using the trapezoid method, approximate the area under $f(x)$ on [0,8] to the nearest integer given the following:

height 1 1 1 2 2 1

x	0	1	2	3	5	7	8
$f(x)$	3.4	2.7	6.2	5.3	1.3	2.1	4.8

Area $=\frac{1}{2}\cdot h$ (sum of bases)

$$=\tfrac{1}{2}(1)(3.4+2.7)+\tfrac{1}{2}(1)(2.7+6.2)+\tfrac{1}{2}(1)(6.2+5.3)+\tfrac{1}{2}(2)(5.3+1.3)+$$
$$\tfrac{1}{2}(2)(1.3+2.1)+\tfrac{1}{2}(1)(2.1+4.8)$$

$$=26.7\approx 27$$

The answer is: 27. Cross out that location number on your clue card and write # 6 as your set.

Treasure Problem: The drop in blood pressure of a typical patient who is given a certain medication is given by $D(x)=.028x^2(19-x)$ where x is the amount of medication in cubic centimeters. What is the maximum drop in blood pressure for this patient to the nearest integer?

x = med amount

D(x) = drop in pressure

① $D(x)=.028x^2(19-x)$

$D(x)=.532x^2-.028x^3$

② $D'(x)=1.064x-.084x^2$

$0=1.064x-.084x^2$

$0=x(1.064-.084x)$

$x=0$

$x=\dfrac{1.064}{.084}=12\frac{2}{3}$

④ $D(12\frac{2}{3})=28.452$

≈ 28

$D(1)$ + $\nearrow$ $D(20)$ +(−) $\searrow$

$12\frac{2}{3}$

③ D increasing to left of $12\frac{2}{3}$, decreasing to right of $12\frac{2}{3}$ so $x=12\frac{2}{3}$ is a max value.

The answer is: 28. Cross out that treasure number on your clue card and write # 6 as your set.

Stu Schwartz

AB Clue Problem Set # 7 Solutions

Suspect Problem: To the nearest integer, find $\int_0^1 \left(4x^{9/5}+15x^{4/3}+13x^{2/3}+14\sin x\right)dx$

$$\boxed{\begin{array}{l}\int_0^1 \left(4x^{9/5}+15x^{4/3}+13x^{2/3}+14\sin x\right)dx\\ \left[\frac{10}{7}x^{14/5}+\frac{45}{7}x^{7/3}+\frac{39}{5}x^{5/3}-14\cos x\right]_0^1\\ 5+\frac{45}{7}+\frac{39}{5}-14\cos(1)+14\approx 22\end{array}}$$

The answer is 22.

Location Problem: Given the following piecewise function, find the value of b that makes the function differentiable.

$$f(x)=\begin{cases}ax^2+10, & x\ge 2\\ x^2-6x+b, x<2\end{cases}$$

$$\boxed{\begin{array}{l}\lim\limits_{x\to 2^+} f(x)=4a+10 \qquad \lim\limits_{x\to 2^-} f(x)=b-8 \qquad \Rightarrow \qquad 4a+10=b-8\Rightarrow b=4a+18\\ f'(x)=\begin{cases}2ax, & x\ge 2\\ 2x-6, x<2\end{cases}\\ \lim\limits_{x\to 2^+} f'(x)=4a \qquad \lim\limits_{x\to 2^-} f(x)=-2 \qquad \Rightarrow \qquad 4a=-2\Rightarrow a=\frac{-1}{2} \qquad \Rightarrow b=4\left(\frac{-1}{2}\right)+18=16\end{array}}$$

The answer is 16.

Treasure Problem: Find the average value of $f(x)=\frac{3\pi}{2}\cos x$ on the interval $\left[0,\frac{\pi}{2}\right]$

$$\boxed{f_{ave}=\frac{3\pi}{2}\frac{\int_0^{\pi/2}\cos x\,dx}{\frac{\pi}{2}-0}=\frac{3\pi}{2}\left[\sin x\right]_0^{\pi/2}\left(\frac{2}{\pi}\right)=3}$$

The answer is 3.

AB Clue Problem Set # 7

Suspect Problem: To the nearest integer, find $\int_0^1 \left(4x^{9/5}+15x^{4/3}+13x^{2/3}+14\sin x\right)dx$

$$4\left(\frac{5}{14}\right)x^{14/5}+15\left(\frac{3}{7}\right)x^{7/3}+13\left(\frac{3}{5}\right)x^{5/3}-14\cos x\Big|_0^1$$

$$\left[\frac{20}{14}+\frac{45}{7}+\frac{39}{5}-14\cos 1\right]-\left[0-14\cos 0\right]$$

$$\approx 22.092 \approx 22$$

The answer is: 22. Cross out that suspect number on your clue card and write # 7 as your set.

Location Problem: Given the following piecewise function, find the value of b that makes the function $\boxed{\text{differentiable}}$ differentiable.

$$f(x)=\begin{cases} ax^2+10, & x\geq 2 \\ x^2-6x+b, x<2 \end{cases}$$

① $\boxed{\text{Continuous}}$

at x=2

$$ax^2+10=x^2-6x+b$$
$$a\cdot 2^2+10=2^2-6(2)+b$$
$$4a+10=4-12+b$$
$$4a+10=-8+b$$
$$4a+18=b$$

② $$f'(x)=\begin{cases} 2ax & x\geq 2 \\ 2x-6 & x<2 \end{cases}$$

$$2\cdot a\cdot 2=2(2)-6$$
$$4a=-2$$
$$a=-2/4=-1/2$$

③ $$4(-1/2)+18=b$$
$$-2+18=b$$
$$\boxed{16=b}$$

The answer is: 16. Cross out that location number on your clue card and write # 7 as your set.

Treasure Problem: Find the average value of $f(x)=\frac{3\pi}{2}\cos x$ on the interval $\left[0,\frac{\pi}{2}\right]$

$$f_{ave}(x)=\frac{3\pi}{2}\left(\frac{\int_0^{\pi/2}\cos x\,dx}{\frac{\pi}{2}-0}\right)=\frac{3\pi}{2}\cdot\frac{2}{\pi}\int_0^{\pi/2}\cos x\,dx$$
$$=3\left[\sin x\Big|_0^{\pi/2}\right]$$
$$=3\left[\sin\tfrac{\pi}{2}-\sin 0\right]$$
$$=3\left[1-0\right]=3$$

The answer is: 3. Cross out that treasure number on your clue card and write # 7 as your set.

Stu Schwartz

AB Clue Problem Set # 8 Solutions

Suspect Problem: Given the following points, estimate $\int_{-1.447}^{2.19} f(x)\,dx$ using the right Riemann sum rounded to the nearest integer.

x	-1.447	-.7196	.0078	.7352	1.4626	2.19
$f(x)$	0	10.538	13.5	3.195	5.993	0

$$\begin{aligned}&\text{base} = \frac{2.19-(-1.447)}{5} = .7274\\ &A = .7274\sum_{i=1}^{5} f(x_i) = .7274(10.538+13.5+3.195+5.933) = 24.161\end{aligned}$$

The answer is 24.

Location Problem: Given $f(x) = x^2 + a$ where a is an integer, find the value of c that satisfies the result of the mean value theorem on [0, 38].

$$\begin{aligned}&\frac{f(38)-f(0)}{38} = f'(c)\\ &\frac{1444+a-a}{38} = 2c\\ &38 = 2c\\ &c = 19\end{aligned}$$

The answer is 19.

Treasure Problem: $f(x) = \dfrac{\sqrt{19x - x^2 - 34}}{e^x}$ has a domain of $[a,b]$. Find $b-a$.

$$\begin{aligned}&19x - x^2 - 34 \geq 0\\ &-(x^2 - 19x + 34) \geq 0\\ &-(x-2)(x-17) \geq 0\\ &x \text{ in } [2,17] \Rightarrow b-a = 15\end{aligned}$$

The answer is 15.

AB Clue Problem Set # 8

Suspect Problem: Given the following points, estimate $\int_{-1.447}^{2.19} f(x)\,dx$ using the right Riemann sum rounded to the nearest integer.

Right

x	-1.447	-.7196	.0078	.7352	1.4626	2.19
$f(x)$	0	10.538	13.5	3.195	5.993	0

$(-.7196 - -1.447)(10.538) + (.0078 - -.7196)(13.5) + (.7352 - .0078)(3.195)$

$+ (1.4626 - .7352)(5.993) + (2.19 - 1.4626)(0)$

$= 7.6653412 + 9.8260 + 2.3240 + 4.3593 + 0$

$= 24.169 \approx 24$

The answer is: 24. Cross out that suspect number on your clue card and write # 8 as your set.

Location Problem: Given $f(x) = x^2 + a$ where a is an integer, find the value of c that satisfies the result of the mean value theorem on [0, 38].

a b

① $f'(c) = 2c$

② $f(0) = 0^2 + a = a$

$f(38) = 38^2 + a$

$= 1444 + a$

③ $\frac{f(b) - f(a)}{b - a} = f'(c)$

$\frac{1444 + a - a}{38 - 0} = 2c$

$\frac{1444}{38} = 2c$

$\frac{38}{2} = \frac{2}{2}c$

$\boxed{19 = c}$

The answer is: 19. Cross out that location number on your clue card and write # 8 as your set.

Treasure Problem: $f(x) = \frac{\sqrt{19x - x^2 - 34}}{e^x}$ has a domain of $[a,b]$. Find $b - a$.

← can't be negative!

never = 0 → d = ℝ

f(0) + f(5) - f(20) +

2 17

domain

[2, 17]

$19x - x^2 - 34 > 0$

$0 > x^2 - 19x + 34$

$0 > (x - 17)(x - 2)$

$6 > x - 17 \quad 0 > x - 2$

$17 > x \quad 2 > x$

$b - a = 17 - 2 = \boxed{15}$

The answer is: 15. Cross out that treasure number on your clue card and write # 8 as your set.

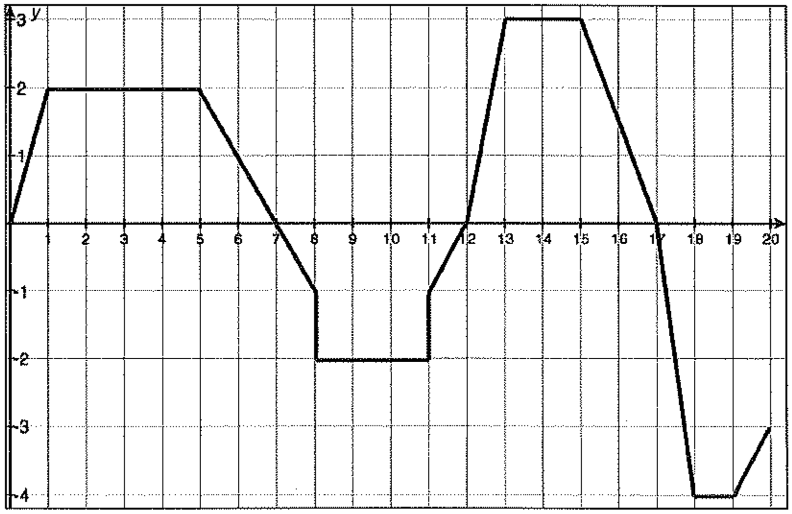
AB Clue Problem Set # 9 Solutions

Suspect Problem: Find $\lim\limits_{x\to\infty}\dfrac{54x^3-5x^2+7x}{\sqrt{9x^6+16x^4+9}}$

$$\boxed{\lim_{x\to\infty}\frac{54x^3-5x^2+7x}{\sqrt{9x^6+16x^4+9}}=\lim_{x\to\infty}\frac{54x^3}{3x^3}=18}$$

The answer is 18.

Location Problem:



Let $F(x)=\int_1^x f(t)\,dt$ where $f(x)$ is the graph above. Find the value of x such that $F(x)=10.5$.

$$\boxed{\begin{array}{l}\int_1^7 f(t)\,dt=10,\int_7^8 f(t)\,dt=-.5,\int_8^{11} f(t)\,dt=-6,\int_{11}^{12} f(t)\,dt=-.5,\int_{12}^{13} f(t)\,dt=1.5,\int_{13}^{15} f(t)\,dt=6\\ 10-.5-6-.5+1.5+6=10.5\Rightarrow\int_1^{15} f(t)\,dt=10.5\Rightarrow F(15)=10.5\end{array}}$$

The answer is 15.

Treasure Problem: A particle is moving along a horizontal line with an acceleration function $a(t)=6t-16$. What is the position s of the particle when it reaches a velocity of 24 given $v(5)=7$ and $s(5)=4$?

$$\boxed{\begin{array}{ll}v(t)=\int a(t)\,dt=3t^2-16t+C_1 & v(5)=3(5)^2-16(5)+C_1=7\Rightarrow C_1=12\\ v(t)=3t^2-16t+12 & 3t^2-16t+12=24\Rightarrow(t-6)(3t+2)=0\Rightarrow t=6\\ s(t)=\int v(t)\,dt=t^3-8t^2+12t+C_2 & s(5)=5^3-8(5)^2+12(5)+C_2=4\Rightarrow C_2=19\\ s(t)=t^3-8t^2+12t+19 & s(6)=6^3-8(6)^2+12(6)+19=19\end{array}}$$

The answer is 19.

AB Clue Problem Set # 9

Suspect Problem: Find $\lim_{x\to\infty}\dfrac{54x^3-5x^2+7x}{\sqrt{9x^6+16x^4+9}}$

Look @ end behavior

$\lim_{x\to\infty} \dfrac{54x^3}{\sqrt{9x^6}} = \lim_{x\to\infty} \dfrac{54\cancel{x^3}}{3\cancel{x^3}}$

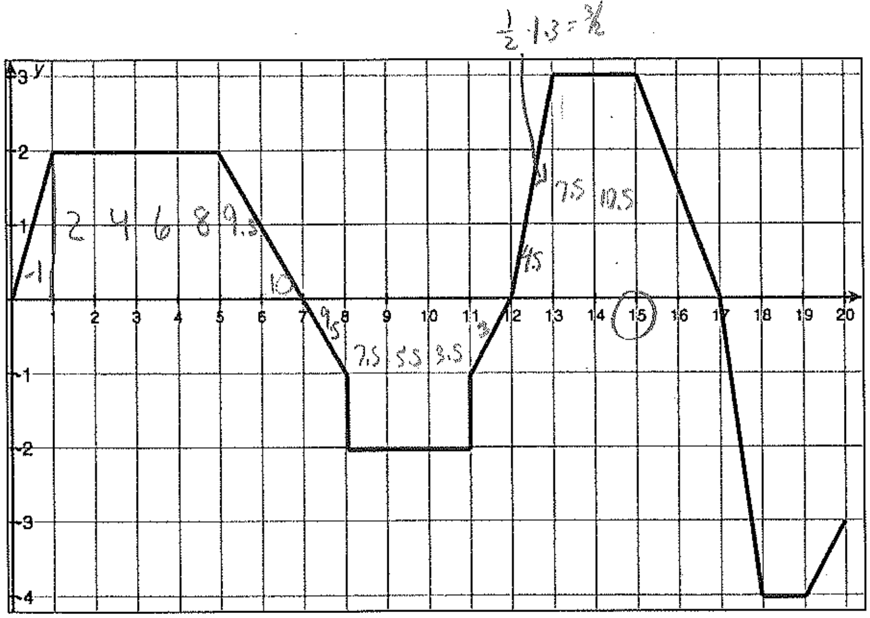
$= \dfrac{54}{3} = 18$

The answer is: 18. Cross out that suspect number on your clue card and write # 9 as your set.

Location Problem:

Count area squares

Start with $F(1) = 0$



Let $F(x)=\int_{①}^{x} f(t)\,dt$ where $f(x)$ is the graph above. Find the value of x such that $F(x)=10.5$.

The answer is: 15. Cross out that location number on your clue card and write # 9 as your set.

Treasure Problem: A particle is moving along a horizontal line with an acceleration function $a(t)=6t-16$. What is the position s of the particle when it reaches a velocity of 24 given $v(5)=7$ and $s(5)=4$?

$a(t) = 6t - 16$

$v(t) = \dfrac{6t^2}{2} - 16t + c$

$7 = 3(5^2) - 16(5) + c$

$7 = 75 - 80 + c$

$7 = -5 + c$

$12 = c$

$v(t) = 3t^2 - 16t + 12$

$s(t) = \dfrac{3t^3}{3} - \dfrac{16t^2}{2} + 12t + c$

$4 = 5^3 - 8(5^2) + 12(5) + c$

$4 = 125 - 200 + 60 + c$

$4 = -15 + c$

$19 = c$

Initial values

$s(t) = t^3 - 8t^2 + 12t + 19$

$24 = 3t^2 - 16t + 12$

$0 = 3t^2 - 16t - 12$

$0 = (3t+2)(t-6)$

$t = -2/3 \quad t = 6$

$s(6) = 6^3 - 8(6^2) + 12(6) + 19$

$= 19$

26
34

The answer is: 19. Cross out that treasure number on your clue card and write # 9 as your set.