AB Clue Problem Set # 10 Solutions

Suspect Problem: A circle has a radius of $\frac{10}{\pi - 1}$ which is the same value as the side of a square. Both the radius of the circle and side of the square are growing at 1 in/sec. Find the difference between the rates of change of their areas in in/sec.

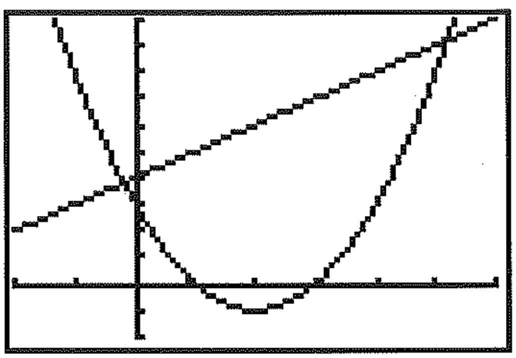
$$A_{\text{circle}} = \pi r^2 \qquad\qquad A_{\text{square}} = r^2$$

$$\frac{dA}{dt} = 2\pi r\frac{dr}{dt} = 2\pi r(1) = 2\pi r \qquad\qquad \frac{dA}{dt} = 2r\frac{dr}{dt} = 2r(1) = 2r$$

$$\text{Difference} = 2\pi r - 2r = 2r(\pi - 1) = 2\left(\frac{10}{\pi - 1}\right)(\pi - 1) = 20$$

The answer is 20.

Location Problem: Find the area bounded by $y = x^2 - 4x + 3$ and $y = x + 4$ to the nearest integer.



$$A = \int_{-.193}^{5.193} \left[x + 4 - \left(x^2 - 4x + 3\right)\right] dx$$

$$A = \int_{-.193}^{5.193} \left(5x + 1 - x^2\right) dx = 26.028$$

The answer is 26.

Treasure Problem: If $f(x) = \int \left(3x^2 + 2x + 4\right) dx$, find C if $f(1) = 10$.

$$f(x) = x^3 + x^2 + 4x + C$$

$$f(1) = 1 + 1 + 4 + C = 10$$

$$C = 4$$

The answer is 4.

AB Clue Problem Set # 10

Suspect Problem: A circle has a radius of $\frac{10}{\pi - 1}$ which is the same value as the side of a square. Both the radius of the circle and side of the square are growing at 1 in/sec. Find the difference between the rates of change of their areas in in/sec.

○ $A = \pi r^2$

$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$

$\frac{dA}{dt} = 2\pi r$

□ $A = \text{side}^2$

$\frac{dA}{dt} = 2r \frac{ds}{dt}$

$\frac{dA}{dt} = 2r$

difference

$2\pi r - 2r$

$= 2r(\pi - 1)$

$= 2\left(\frac{10}{\cancel{\pi - 1}}\right)\left(\frac{\cancel{\pi - 1}}{1}\right)$

$= 20$ in/sec

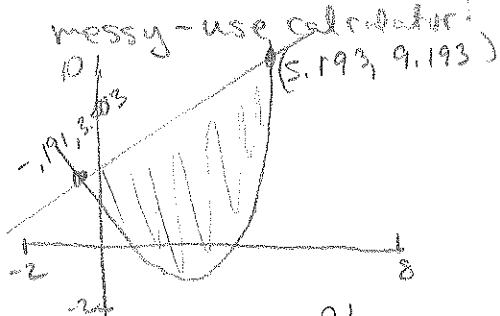
The answer is: 20. Cross out that suspect number on your clue card and write # 10 as your set.

Location Problem: Find the area bounded by $y = x^2 - 4x + 3$ and $y = x + 4$ to the nearest integer.

find limits $x^2 - 4x + 3 = x + 4$

$x^2 - 5x - 1 = 0$

messy - use calculator!



$$\int_{-1.913}^{5.193} (x+4) - (x^2 - 4x + 3)\, dx \approx 26.028$$

- Store limits in calculator
- write integral as shown
- calculate
- write answer

The answer is: 26. Cross out that location number on your clue card and write # 10 as your set.

Treasure Problem: If $f(x) = \int (3x^2 + 2x + 4)\, dx$, find C if $f(1) = 10$.

$f(x) = \frac{3x^3}{3} + \frac{2x^2}{2} + 4x + C$

$10 = 1^3 + 1^2 + 4(1) + C$

$10 = 6 + C$

$4 = C$

The answer is: 4. Cross out that treasure number on your clue card and write # 10 as your set.

AB Clue Problem Set # 11 Solutions

Suspect Problem: Let $f(x)=\dfrac{17x}{e^{3x}}$. Find the slope of the tangent line to f at $x=0$.

$$\boxed{f'(x)=\frac{e^{3x}(17)-17x\left(3e^{3x}\right)}{e^{6x}}\Rightarrow f'(0)=\frac{17-0}{1}=17}$$

The answer is 17.

Location Problem: Find the value of b such that the average value of $f(x)=3x^2-6x-12$ on $[0,b]$ is -12.

$$f_{ave}=\frac{\int_0^b\left(3x^2-6x-12\right)dx}{b}=-12\Rightarrow\frac{\left[x^3-3x^2-12x\right]_0^b}{b}=-12$$

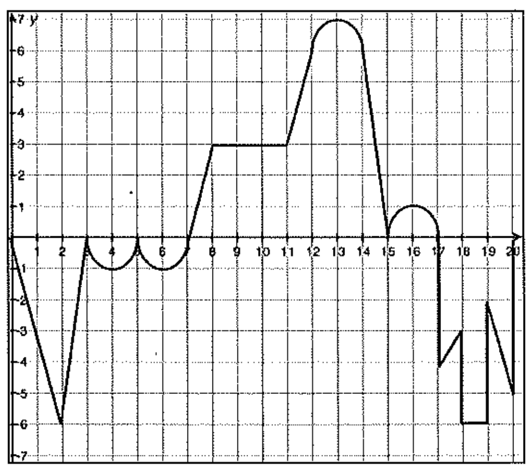
$$\frac{b^3-3b^2-12b}{b}=-12\Rightarrow b^2-3b-12=-12$$

$$b(b-3)=0\Rightarrow b=3$$

The answer is 3.

Treasure Problem:

Let $F(x)=\int_0^x f(t)\,dt$ where f is graphed to the right (consisting of lines and semicircles). Find $F(20)$.



$\int_0^3 f(t)\,dt=-9$	$\int_3^7 f(t)\,dt=-\pi$	$\int_7^{11} f(t)\,dt=10.5$	$\int_{11}^{12} f(t)\,dt=4.5$
$\int_{12}^{14} f(t)\,dt=12+\frac{\pi}{2}$	$\int_{14}^{15} f(t)\,dt=3$	$\int_{15}^{17} f(t)\,dt=\frac{\pi}{2}$	$\int_{17}^{18} f(t)\,dt=-3.5$
$\int_{18}^{19} f(t)\,dt=-6$	$\int_{19}^{20} f(t)\,dt=-3.5$	$\int_0^{20} f(t)\,dt=8$	

The answer is 8.

AB Clue Problem Set # 11

Suspect Problem: Let $f(x)=\dfrac{17x}{e^{3x}}$. Find the slope of the tangent line to f at $x=0$.

$$f'(x)=\frac{e^{3x}(17)-17x(3e^{3x})}{(e^{3x})^2}$$

$$f'(0)=\frac{17(e^0)-0(3e^0)}{(e^0)^2}=\frac{17}{1}=17$$

The answer is: 17. Cross out that suspect number on your clue card and write # 11 as your set.

Location Problem: Find the value of b such that the average value of $f(x)=3x^2-6x-12$ on $[0,b]$ is -12.

① $\dfrac{1}{b-0}\displaystyle\int_0^b (3x^2-6x-12)\,dx=-12$

$\dfrac{1}{b}\left[\dfrac{\cancel{3}x^3}{\cancel{3}}-\dfrac{6x^2}{2}-12x\right]_0^b=-12$

$x^3-3x^2-12x=-12b$

$(b^3-3b^2-12b)-0=-12b$

$b^3-3b^2-\cancel{12b}=-\cancel{12b}$

$b^3-3b^2=0$

$b^2(b-3)=0$

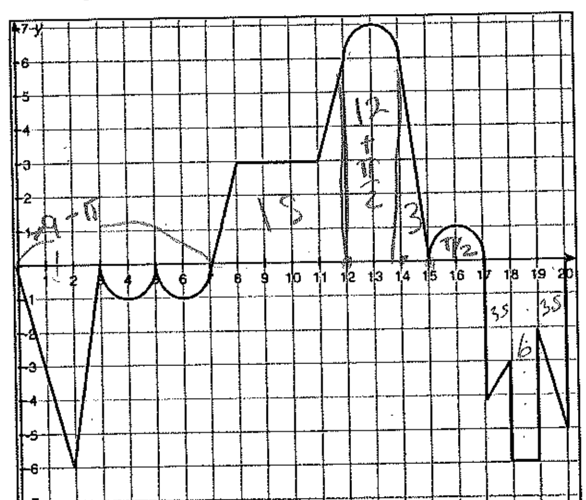
$b=0$ $b=3$

endpt   answer

The answer is: 3. Cross out that location number on your clue card and write # 11 as your set.

Treasure Problem:

Let $F(x)=\displaystyle\int_0^x f(t)\,dt$ where f is graphed to the right (consisting of lines and semicircles). Find $F(20)$.



$F(3)=\frac{1}{2}(3)(-6)=-9$

$\int_3^5 f(t)\,dt = \frac{1}{2}\cdot 1^2\pi=-\frac{\pi}{2}$

$\int_5^7 f(t)\,dt=-\pi/2$

$\int_0^7 f(t)\,dt=-9-\pi=-(9+\pi)$

$\int_7^{12} f(t)\,dt=12+\frac{3}{2}+\frac{3}{2}=15$

$\int_{12}^{14} f(t)\,dt=12+\frac{\pi}{2}$

16.429

$\int_{14}^{15} f(t)\,dt=\frac{1}{2}\cdot 1.6=3$

$\int_{15}^{17}=\frac{\pi}{2}$

$\int_{?}^{8} f(t)\,dt=\frac{1}{2}\cdot 1.3=\frac{3}{2}$

$F(17)=31$

$F(20)=-9-\cancel{\pi}+15+12+\frac{\pi}{2}+3+\frac{\pi}{2}-7-6=$

$-22+30=8$

The answer is: 8. Cross out that treasure number on your clue card and write # 11 as your set.

Stu Schwartz

AB Clue Problem Set # 12 Solutions

Suspect Problem: People are entering a zoo at the rate of $100e^t + 75t$ people per hour where t is the amount of time the zoo has been open on that day measured in hours. If the doors are open at 9:00 AM, how many hundreds of people have entered the zoo at 11:40 AM? (nearest integer).

$$P = \int_0^{2.667} \left(100e^t + 75t\right) dt$$

$$P = \left[100e^t + \frac{75t^2}{2}\right]_0^{2.667}$$

$$P = 100e^{2.667} + \frac{75(2.667)^2}{2} - 100 \approx 1606$$

The answer is 16.

Location Problem: Find $\lim\limits_{x\to 0} \frac{7x}{\sqrt{x+4}-2}$

$$\lim_{x\to 0}\left(\frac{7x}{\sqrt{x+4}-2}\right)\left(\frac{\sqrt{x+4}+2}{\sqrt{x+4}+2}\right)$$

$$\lim_{x\to 0}\frac{7x\left(\sqrt{x+4}+2\right)}{x+4-4} = \lim_{x\to 0} 7\left(\sqrt{x+4}+2\right) = 7(2+2) = 28$$

The answer is 28.

Treasure Problem: Find the average value of $f(x)$ in the interval [-1, 3] when $f'(x) = 3x^2 - 6x$ and $f(2) = 0$.

$$f(x) = \int \left(3x^2 - 6x\right) dx = x^3 - 3x^2 + C$$

$$f(2) = 8 - 12 + C = 0 \Rightarrow C = 4$$

$$f(x) = x^3 - 3x^2 + 4$$

$$f_{ave} = \frac{\int_{-1}^{3}\left(x^3 - 3x^2 + 4\right) dx}{4} = \left[\frac{\frac{x^4}{4} - x^3 + 4x}{4}\right]_{-1}^{3} = \frac{\frac{81}{4} - 27 + 12 - \left(\frac{1}{4} + 1 - 4\right)}{4} = 2$$

The answer is 2.

 Stu Schwartz

AB Clue Problem Set # 12

Suspect Problem: People are entering a zoo at the rate of $100e^t + 75t$ people per hour where t is the amount of time the zoo has been open on that day measured in hours. If the doors are open at 9:00 AM, how many $\boxed{\text{hundreds of people}}$ have entered the zoo at 11:40 AM? (nearest integer).

$$P = \int_0^{2\,2/3} (100e^t + 75t)\,dt = 100e^t + \frac{75t^2}{2}\Big|_0^{2\,2/3}$$

$$P = \left[100e^{8/3} + \frac{75}{2}\left(\frac{8}{3}\right)^2\right] - \left[100e^0 + \frac{75}{2}(0^2)\right]$$

$$P = 1705.858 - 100 - 0 = 1605.858 \approx 1606 \text{ people}$$

$$\frac{1606}{100} = 16 \text{ hundred people}$$

The answer is: 16. Cross out that suspect number on your clue card and write # 12 as your set.

Location Problem: Find $\lim\limits_{x \to 0} \dfrac{7x}{\sqrt{x+4}-2}$ $= \dfrac{7 \cdot 0}{\sqrt{0+4}-2}$ no!

use complex conjugate

$$\lim_{x \to 0} \frac{7x}{\sqrt{x+4}-2} \cdot \frac{\sqrt{x+4}+2}{\sqrt{x+4}+2} = \lim_{x \to 0} \frac{7x(\sqrt{x+4}+2)}{x+4-4}$$

$$= \lim_{x \to 0} \frac{7\cancel{x}(\sqrt{x+4}+2)}{\cancel{x}} = 7(\sqrt{0+4}+2) = 7(2+2) = 7 \cdot 4 = \boxed{28}$$

The answer is: 28. Cross out that location number on your clue card and write # 12 as your set.

Treasure Problem: Find the average value of $f(x)$ in the interval [-1, 3] when $f'(x) = 3x^2 - 6x$ and $f(2) = 0$.

Find $f(x)$

$$f(x) = \int (3x^2 - 6x)\,dx \qquad f(x) = x^3 - 3x^2 + 4$$

$$= x^3 - 3x^2 + C$$

$$0 = 2^3 - 3 \cdot 2^2 + C$$

$$0 = 8 - 12 + C$$

$$0 = -4 + C$$

$$4 = C$$

Find average value

$$\frac{1}{3-(-1)} \int_{-1}^{3} (x^3 - 3x^2 + 4)\,dx = \frac{1}{4}\left[\frac{x^4}{4} - x^3 + 4x \Big|_{-1}^{3}\right]$$

$$= \frac{1}{4}\left[\left(\frac{3^4}{4} - 3^3 + 4(3)\right) - \left(\frac{(-1)^4}{4} - (-1)^3 + 4(-1)\right)\right]$$

$$= \frac{1}{4}\left[\left(\frac{81}{4} - 27 + 12\right) - \left(\frac{1}{4} + 1 - 4\right)\right] = \frac{1}{4}\left[\left(20\tfrac{1}{4} - 15\right) - \left(-2\tfrac{3}{4}\right)\right]$$

$$= \frac{1}{4}\left[5\tfrac{1}{4} + 2\tfrac{3}{4}\right] = \frac{1}{4}[8] = 2$$

The answer is: 2. Cross out that treasure number on your clue card and write # 12 as your set.

Stu Schwartz

AB Clue Problem Set # 13 Solutions

Suspect Problem: Find the value of c in the interval [1, 5] for which Rolle's Theorem can be applied to $f(x)=3x^2-18x+15$

$$\boxed{\begin{array}{l} f \text{ is continuous and differentiable and } f(1)=f(5)=0 \\ f'(x)=6x-18=0 \\ 6x=18 \Rightarrow x=3 \end{array}}$$

The answer is 3.

Location Problem: Three months after it stopped advertising. a computer company noticed that its sales proceeds had dropped from \$39 million per month to \$27.89 million per month. If the sales prices follow an exponential pattern of decline, what will be the proceeds in another three months to the nearest million?

$$\boxed{\begin{array}{l} P=Ce^{kt} \Rightarrow 39=Ce^{0} \Rightarrow C=39 \\ P=39e^{kt} \Rightarrow 27.89=39e^{3k} \Rightarrow e^{3k}=\dfrac{27.89}{39} \Rightarrow k=\dfrac{\ln\left(\dfrac{27.89}{39}\right)}{3}\approx -.1118 \\ P=39e^{6(-.1118)}\approx 19.94 \end{array}}$$

The answer is 20.

Treasure Problem: Find the value of k to the nearest integer such that the line $x=k$ divides the area under $f(x)=\dfrac{x^3}{36}-x+15$ on $[0,20]$ into two equal areas.

$$\boxed{\begin{array}{l} 2\displaystyle\int_0^k\left(\frac{x^3}{36}-x+15\right)dx=\int_0^{20}\left(\frac{x^3}{36}-x+15\right)dx \\ 2\left[\dfrac{x^4}{144}-\dfrac{x^2}{2}+15x\right]_0^k=\left[\dfrac{x^4}{144}-\dfrac{x^2}{2}+15x\right]_0^{20} \\ 2\left(\dfrac{k^4}{144}-\dfrac{k^2}{2}+15k\right)=\left(\dfrac{160000}{144}-\dfrac{400}{2}+300\right) \\ \dfrac{k^4}{72}-k^2+30k=\dfrac{10000}{9}+100 \Rightarrow k\approx 16.331 \end{array}}$$

The answer is 16.

 Stu Schwartz

AB Clue Problem Set # 13

Suspect Problem: Find the value of c in the interval [1, 5] for which Rolle's Theorem can be applied to $f(x)=3x^2-18x+15$

a b

same as mean Value thm!

Rolle's thm: f is continuous & differentiable

$\frac{f(b)-f(a)}{b-a}=f'(c)$

$f(5)=3\cdot 5^2-18(5)+15$
$=75-90+15$
$=0$

$f(1)=3(1^2)-18(1)+15$
$=0$

$\frac{0-0}{5-1}=6c-18$

$0=6c-18$

$\frac{18}{6}=c$

$\boxed{3=c}$

The answer is: 3. Cross out that suspect number on your clue card and write # 13 as your set.

Location Problem: Three months after it stopped advertising. a computer company noticed that its sales proceeds had dropped from \$39 million per month to \$27.89 million per month. If the sales prices follow an exponential pattern of decline, what will be the proceeds in another three months to the nearest million?

(0, 39) ← A_0
(3, 27.89)
t P

Profit $= A_0 e^{kt}$

$27.89=39e^{3k}$

$\ln\left(\frac{27.89}{39}\right)=\cancel{\ln e}^{3k}$

$\frac{\ln\left(\frac{27.89}{39}\right)}{3}=k$

$-0.1118\approx k$

$P=39e^{-0.1118t}$

$P=39e^{(-0.1118\cdot 6)}$

$P=19.945$ million

$\boxed{P\approx 20}$

The answer is: 20. Cross out that location number on your clue card and write # 13 as your set.

Treasure Problem: Find the value of k to the nearest integer such that the line $x=k$ divides the area under $f(x)=\frac{x^3}{36}-x+15$ on $[0,20]$ into two equal areas.

$2\int_0^k\left(\frac{x^3}{36}-x+15\right)dx=\int_0^{20}\left(\frac{x^3}{36}-x+15\right)dx$

$2\left[\frac{x^4}{144}-\frac{x^2}{2}+15x\Big|_0^k\right]=\frac{x^4}{144}-\frac{x^2}{2}+15x\Big|_0^{20}$

$2\left[\frac{k^4}{144}-\frac{k^2}{2}+15k\right]=\frac{20^4}{144}-\frac{20^2}{2}+15(20)$

$2\left[\frac{k^4}{144}-\frac{k^2}{2}+15k\right]=\left(\frac{160000}{144}-200+300\right)$

$\frac{k^4}{72}-k^2+30k=\frac{10000}{9}+100$

graph each side
calc, intersect
$k\approx 16.331$

The answer is: 16. Cross out that treasure number on your clue card and write # 13 as your set.

Stu Schwartz

AB Clue Problem Set # 14 Solutions

Suspect Problem: Find the larger of two numbers whose sum is 30 for which the sum of their squares is a minimum.

$$\begin{array}{|ll|}\hline S=x^2+y^2 & x+y=30 \\ S=x^2+(30-x)^2 & y=30-x \\ S'=2x+2(30-x)(-1)=0 \Rightarrow 4x-60=0 & \\ x=15 & \\ \hline\end{array}$$

The answer is 15.

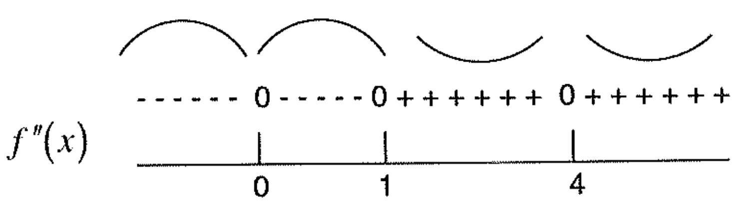
Location Problem: Find the only value not included in the range of $y=14-\dfrac{1}{x}$.

$$\begin{array}{|l|}\hline y=14-\dfrac{1}{x} \\ xy=14x-1 \\ 1=14x-xy \\ \dfrac{1}{14-y}=x \Rightarrow y\neq 14 \\ \hline\end{array}$$

The answer is 14.

Treasure Problem: How many inflection points are in the graph of $f(x)=\dfrac{x^7}{42}-\dfrac{3x^6}{10}+\dfrac{6x^5}{5}-\dfrac{4x^4}{3}$?

$$\begin{array}{|l|}\hline f'(x)=\dfrac{x^6}{6}-\dfrac{9x^5}{5}+6x^4-\dfrac{16x^3}{3} \\ f''(x)=x^5-9x^4+24x^3-16x^2=0 \\ x^2\left(x^3-9x^2+24x-16\right)=0 \\ x^2(x-1)(x-4)^2=0 \\ \text{Only point of inflection is at } x=1 \\ \hline\end{array}$$



The answer is 1.

Stu Schwartz

AB Clue Problem Set # 14

Suspect Problem: Find the larger of two numbers whose sum is 30 for which the sum of their squares is a minimum.

$x+y=30 \rightarrow y=30-x$

minimize $f(x)=x^2+y^2$

$f(x)=x^2+(30-x)^2$

$f(x)=x^2+900-60x+x^2$

$f(x)=900-60x+2x^2$

$f'(x)=-60+4x$

$0=-60+4x$

$15=x$

$f'(0)$ $f'(20)$

15

f(x) is decreasing to left of x=15 and increasing to right so x=15 is a minimum value.

$x=15,\ y=15$

The answer is: 15. **Cross out that suspect number on your clue card and write # 14 as your set.**

Location Problem: Find the only value not included in the range of $y=14-\dfrac{1}{x}$.

Domain is $(-\infty,0)\cup(0,\infty)$

So Range is $\boxed{\mathbb{R} \neq 14}$

answer

$\boxed{\text{Reason}}$

b/c $-\frac{1}{x}$ always changes 14 and can never = 0

Algebraically

$y=14-\frac{1}{x}$

$xy=14x-1$

$1=14x-xy$

$1=x(14-y)$

$\dfrac{1}{14-y}$

The answer is: 14. **Cross out that suspect number on your clue card and write # 14 as your set.**

Treasure Problem: How many inflection points are in the graph of $f(x)=\dfrac{x^7}{42}-\dfrac{3x^6}{10}+\dfrac{6x^5}{5}-\dfrac{4x^4}{3}$?

$f'(x)=\dfrac{7x^6}{42}-6\left(\dfrac{3x^5}{10}\right)+5\left(\dfrac{6x^4}{5}\right)-4\left(\dfrac{4x^3}{3}\right)$

$f'(x)=\dfrac{x^6}{6}-\dfrac{9x^5}{5}+6x^4-\dfrac{16x^3}{3}$

$f''(x)=6\left(\dfrac{x^5}{6}\right)-5\left(\dfrac{9x^4}{5}\right)+4(6x^3)-3\left(\dfrac{16}{3}x^2\right)$

$f''(x)=x^5-9x^4+24x^3-16x^2$

$0=x^2(x^3-9x^2+24x-16)$

inflection pts where $f''(x)=0$ and changes sign

Calc zeroes $x=1$ $x=4$

$-\ -\ 0\ -\ -\ 0\ +\ +\ 0\ +\ +$

0 1 4

only pt of inflection is at $x=1$

The answer is: 1. **Cross out that suspect number on your clue card and write # 14 as your set.**

Stu Schwartz

AB Clue Problem Set # 15 Solutions

Suspect Problem: Find $\lim\limits_{x \to -2} \dfrac{60x+120}{2x^3+6x^2+2x-4}$

$$\lim_{x \to -2} \frac{60(x+2)}{2\left(x^3+3x^2+x-2\right)}$$

$$\lim_{x \to -2} \frac{60(x+2)}{2(x+2)\left(x^2+x-1\right)}$$

$$\lim_{x \to -2} \frac{60}{2\left(x^2+x-1\right)} = \frac{30}{4-2-1} = 30$$

The answer is 30.

Location Problem: Snow falls intermittently accumulating on the ground at a rate (inches/hour) given by the equation $f(t) = t^2 \sin t^3 + 2.5$ where t is the number of hours that storm is overhead. To the nearest inch, how much snow will accumulate in the first two hours of the storm?

$$R = \int_0^2 \left(t^2 \sin t^3 + 2.5\right) dt \qquad u = t^3, du = 3t^2 dt$$

$$R = \frac{1}{3}\int_0^8 \sin u \, du + \int_0^2 2.5 \, dt \qquad t=0, u=0, t=2, u=8$$

$$R = \frac{-1}{3}\left[\cos u\right]_0^8 + \left[2.5t\right]_0^2$$

$$R = \frac{-1}{3}\cos 8 + \frac{1}{3} + 5 = 5.381$$ (Note that in the interval $[0,2]$ there is a time when the snow is actually melting!)

The answer is 5.

Treasure Problem: A particle moves along the x-axis with a velocity given by $v(t) = \frac{-1}{3}t^3 + 2t^2 + 18t$. What is the maximum acceleration of the particle on the interval [0, 4] ?

$$v(t) = \frac{-1}{3}t^3 + 2t^2 + 18t$$

$$a(t) = -t^2 + 4t + 18$$

$$a'(t) = -2t + 4 = 0 \Rightarrow t = 2$$

$$a(2) = -4 + 8 + 18 = 22$$

$a'(t)$	+ + + + + + + + + + + 0	- - - - - - - - - - - - - - - - - -
	0	2	4

The answer is 22.

 Stu Schwartz

AB Clue Problem Set # 15

Suspect Problem: Find $\lim_{x \to -2} \dfrac{60x+120}{2x^3+6x^2+2x-4}$ $= \lim_{x \to -2} \dfrac{60(x+2)}{2(x^3+3x^2+x-2)}$

$$
\begin{array}{r}
x^2+x-1 \\
x+2 \overline{)\, x^3+3x^2+x-2} \\
-(x^3+2x^2) \\
\hline
x^2+x \\
-(x^2+2x) \\
\hline
-x-2 \\
-x-2 \\
\hline
0
\end{array}
$$

$= \lim_{x \to -2} \dfrac{30(\cancel{x+2})}{(\cancel{x+2})(x^2+x-1)}$

$= \dfrac{30}{(-2)^2+(-2)-1} = \dfrac{30}{4-3} = \boxed{30}$

The answer is: 30. **Cross out that suspect number on your clue card and write # 15 as your set.**

Location Problem: Snow falls intermittently accumulating on the ground at a rate (inches/hour) given by the equation $f(t)=t^2\sin t^3+2.5$ where t is the number of hours that storm is overhead. To the nearest inch, how much snow will accumulate in the first two hours of the storm?

$u = t^3 \qquad u(0)=0 \quad u(2)=8$

$du = 3t^2\,dt$

$\int_0^2 t^2\sin(t^3)+2.5\,dt$

$= \frac{1}{3}\int_0^2 3t^2\sin t^3\,dt + \int_0^2 2.5\,dt$

$= \frac{1}{3}\int_0^8 \sin u\,du + \int_0^2 2.5\,dt$

$= \frac{1}{3}\left[-\cos u\Big|_0^8\right] + 2.5t\Big|_0^2$

$-\frac{1}{3}[\cos 8 - \cos 0] + 2.5(2)$

$= 5.382$

$\approx \boxed{5 \text{ inches}}$

The answer is: 5. **Cross out that location number on your clue card and write # 15 as your set.**

Treasure Problem: A particle moves along the x-axis with a velocity given by $v(t)=\dfrac{-1}{3}t^3+2t^2+18t$. What is the maximum acceleration of the particle on the interval $[0, 4]$?

1. $a(t) = -\frac{1}{\cancel{3}}\cancel{3}t^2 + 2(2t)+18$

$a(t) = -t^2+4t+18$

2. $a'(t) = -2t+4$

$0 = -2t+4$

$2t = 4$

$t = 2$

$a'(0)=4, \quad a'(3)=-2$

Since a' is positive to left of $x=2$ and negative to right of $x=2$, $x=2$ is a maximum

Find max acceleration

$a(2) = -(2^2)+4(2)+18$

$= -4+8+18$

$= 22$

The answer is: 22. **Cross out that treasure number on your clue card and write # 15 as your set.**

Stu Schwartz

AB Clue Problem Set # 16 Solutions

Suspect Problem: The acceleration of an object is given by the function $a(t)=\frac{-t}{2}+\frac{9}{4}$. Also, at time $t=0$, the velocity of the object is -2. Find the difference between the distance and the displacement traveled by the object to the nearest integer from $t=0$ to $t=10$.

$$\begin{aligned}
v(t) &= \int\left(\frac{-t}{2}+\frac{9}{4}\right)dt\\
v(t) &= \frac{-t^2}{4}+\frac{9}{4}t+C\\
v(0) &= C=-2\\
v(t) &= \frac{-t^2}{4}+\frac{9}{4}t-2
\end{aligned}$$

$$\text{displacement} = \int_0^{10} v(t)\,dt = \int_0^{10}\left(\frac{-t^2}{4}+\frac{9}{4}t-2\right)dt = \left[\frac{-t^3}{12}+\frac{9t^2}{8}-2t\right]_0^{10} = 9.167$$

$$\text{distance} = \int_0^{10}|v(t)|\,dt = \int_0^{1} v(t)\,dt + \int_8^{10} v(t)\,dt - \int_1^{8} v(t)\,dt = 19.417$$

$$\text{difference} = 19.417-9.167=10.25\approx 10$$

The answer is 10.

Location Problem: $f(x)=\begin{cases} ax^2+1, x\ge 1\\ bx-3, x<1\end{cases}$ If $f(x)$ is differentiable, find the value of $\frac{b}{2a}$.

$$\lim_{x\to 1^+} f(x)=a+1 \qquad \lim_{x\to 1^-} f(x)=b-3 \Rightarrow a=b-4$$

$$f'(x)=\begin{cases} 2ax^2, x\ge 1\\ b, x<1\end{cases} \qquad \lim_{x\to 1^+} f'(x)=2a \qquad \lim_{x\to 1^-} f'(x)=b \Rightarrow 2a=b$$

$$a=2a-4 \Rightarrow a=4, b=8 \Rightarrow \frac{b}{2a}=1$$

The answer is 1.

Treasure Problem: Given that $f(x)=x^2-5$ on the interval [0, 50.2], find the value of c to the nearest integer guaranteed by the mean value theorem for integrals.

$$\begin{aligned}
f(c) &= \frac{\int_0^{50.2}\left(x^2-5\right)dx}{50.2}\\
f(c) &= \frac{41917.669}{50.2}=835.013\\
c^2-5 &= 835.013 \Rightarrow c^2=840.013\\
c &= 28.983\approx 29
\end{aligned}$$

The answer is 29.

AB Clue Problem Set # 16

Suspect Problem: The acceleration of an object is given by the function $a(t)=\frac{-t}{2}+\frac{9}{4}$. Also, at time $t=0$, the velocity of the object is -2. Find the difference between the distance and the displacement traveled by the object to the nearest integer from $t=0$ to $t=10$.

$a(t)=-\frac{t}{2}+\frac{9}{4}$

$v(t)=-\frac{t^2}{4}+\frac{9}{4}t+C$

$v_0=-2$ so

$v(t)=-\frac{t^2}{4}+\frac{9}{4}t-2$

$0=-\frac{t^2}{4}+\frac{9}{4}t-2$

$0=-t^2+9t-8$

$0=t^2-9t+8$

$0=(t-8)(t-1)$

$t=8,\ t=1$

test before by (-)

displacement $\int_0^{10}\left(-\frac{t^2}{4}+\frac{9}{4}t-2\right)dt=-\frac{t^3}{12}+\frac{9t^2}{8}-2t\Big|_0^{10}$

$=-\frac{10^3}{12}+\frac{9(10^2)}{8}-2(10)=\frac{55}{6}\approx 9.167$

distance traveled $\int_0^{10}\left|-\frac{t^2}{4}+\frac{9}{4}t-2\right|dt$ $\quad \int_1^{8}-\frac{t^2}{4}+\frac{9}{4}t-2\,dt-\int_8^{10}-\frac{t^2}{4}+\frac{9}{4}t-2\,dt$

$-\quad+\quad-$ on $0\quad 1\quad 8\quad 10$

$=-\int_0^1 -\frac{t^2}{4}+\frac{9}{4}t-2\,dt+$

$=\frac{23}{24}+\frac{343}{24}+\frac{100}{24}=\frac{466}{24}=19.417$

difference $19.417-9.167=10.250$ $\quad$ Answer 10

The answer is: 10. Cross out that suspect number on your clue card and write # 16 as your set.

Location Problem: $f(x)=\begin{cases}ax^2+1, x\geq 1\\ bx-3, x<1\end{cases}$ If $f(x)$ is differentiable, find the value of $\frac{b}{2a}$.

$a(1^2)+1=b(1)-3$

$a+1=b-3$

$a+4=b$

$f'(x)=\begin{cases}2ax & x\geq 1\\ b & x<1\end{cases}$

$2a=b \rightarrow 2(4)=b$

$8=b$

$a+4=2a$

$4=a$

$\frac{b}{2a}=\frac{8}{2(4)}=\frac{8}{8}=1$

The answer is: 1. Cross out that location number on your clue card and write # 16 as your set.

Treasure Problem: Given that $f(x)=x^2-5$ on the interval [0, 50.2], find the value of c to the nearest integer guaranteed by the mean value theorem for integrals.

$\boxed{f(c)}=\frac{1}{50.2}\int_0^{50.2}(x^2-5)\,dx=\frac{1}{50.2}\left[\frac{x^3}{3}-5x\Big|_0^{50.2}\right]$

$=\frac{1}{50.2}\left[\frac{50.2^3}{3}-50.2(5)-0\right]=\frac{1}{50.2}[41917.669]=835.013$

$f(c)=c^2-5$

$835.013=c^2-5$

$\sqrt{840.013}=\sqrt{c^2}$

$c\approx 28.983\approx 29$

The answer is: 29. Cross out that treasure number on your clue card and write # 16 as your set.

Stu Schwartz

AB Clue Problem Set # 17 Solutions

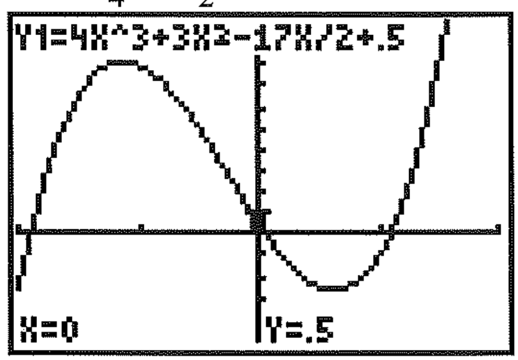
Suspect Problem: A particle is moving along a straight line with position function $s(t) = \tan^{-1} t - \ln t$. To the nearest integer what is the particle's acceleration at $t = 0.387$?

$$s(t) = \tan^{-1} t - \ln t$$
$$v(t) = \frac{1}{1+t^2} - \frac{1}{t}$$
$$a(t) = \frac{-2t}{\left(1+t^2\right)^2} + \frac{1}{t^2}$$
$$a(.387) = \frac{-.774}{\left[1+(.387)^2\right]^2} + \frac{1}{(.387)^2} = 6.091 \approx 6$$

The answer is 6.

Location Problem: What is the maximum value of $f(x) = x^4 + x^3 - \frac{17}{4}x^2 + \frac{1}{2}x$ on the interval [-2, 2] ?

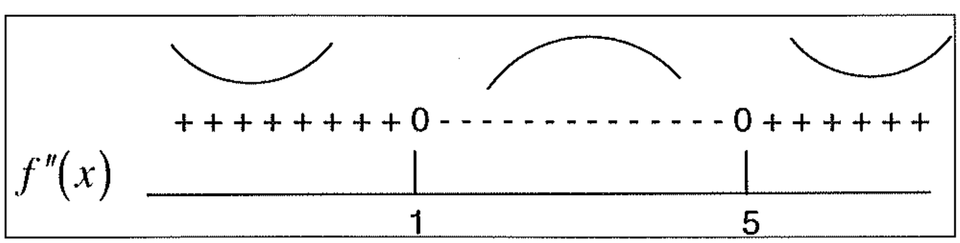
$$f'(x) = 4x^3 + 3x^2 - \frac{17}{2}x + \frac{1}{2} = 0$$
$$x = -1.902, 0.062, 1.092$$
$$f(-1.902) = -10.119$$
$$f(0.062) = 0.015$$
$$f(1.092) = -1.797$$
$$f(-2) = -10$$
$$f(2) = 8$$



The answer is 8.

Treasure Problem: Find the sum of the x-values of the inflection points of $f(x) = \frac{x^4}{12} - x^3 + \frac{5x^2}{2} + x\sqrt{7} + 9$.

$$f'(x) = \frac{x^3}{3} - 3x^2 + 5x + \sqrt{7}$$
$$f''(x) = x^2 - 6x + 5x = 0$$
$$(x-1)(x-5) = 0 \Rightarrow x = 1, x = 5$$
$$1 + 5 = 6$$



The answer is 6.

AB Clue Problem Set # 17

Suspect Problem: A particle is moving along a straight line with position function $s(t) = \tan^{-1} t - \ln t$. To the nearest integer, what is the particle's acceleration at $t = 0.387$?

$v(t) = \frac{1}{1+t^2} - \frac{1}{t} = (1+t^2)^{-1} - t^{-1}$

$a(t) = -1(1+t^2)^{-2}(2t) - (-1 \cdot t^{-2})$

$= \frac{-2t}{(1+t^2)^2} + \frac{1}{t^2}$

$a(0.387) = \frac{-2(0.387)}{(1+0.387^2)^2} + \frac{1}{(0.387)^2}$

$a(0.387) \approx 6.091$

The answer is: 6. Cross out that suspect number on your clue card and write # 17 as your set.

Location Problem: What is the maximum value of $f(x) = x^4 + x^3 - \frac{17}{4}x^2 + \frac{1}{2}x$ on the interval [-2, 2] ?

$f'(x) = 4x^3 + 3x^2 - \frac{17}{2}x + \frac{1}{2}$

$0 = 4x^3 + 3x^2 - \frac{17}{2}x + \frac{1}{2}$

calc zeroes!

$-1.902 \quad .0602 \quad 1.092$

$-2 \qquad\qquad\qquad\qquad 2$

check values w/ $f(x)$ endpts too!

$f(-2) \approx -10$

$f(-1.902) \approx -10.12$

$f(.0602) \approx 0.1493$

$f(1.092) \approx -1.798$

$\boxed{f(2) \approx 8}$

The answer is: 8. Cross out that location number on your clue card and write # 17 as your set.

Treasure Problem: Find the sum of the x-values of the inflection points of $f(x) = \frac{x^4}{12} - x^3 + \frac{5x^2}{2} + x\sqrt{7} + 9$.

f''

$f'(x) = \frac{4x^3}{12} - 3x^2 + 2\left(\frac{5x^1}{2}\right) + \sqrt{7}$

$f'(x) = \frac{x^3}{3} - 3x^2 + 5x + \sqrt{7}$

$f''(x) = x^2 - 6x + 5$

$0 = (x-5)(x-1)$

$x = 5 \quad x = 1$

$f''(0) \quad f''(2) \quad f''(6)$

$+ \quad - \quad +$

$1 \qquad 5$

Inflectn pts at $x = 1$, $x = 5$

$\boxed{\text{Sum } 1+5 = 6}$

The answer is: 6. Cross out that treasure number on your clue card and write # 17 as your set.

 Stu Schwartz

AB Clue Problem Set # 18 Solutions

Suspect Problem: Find the derivative of $y = \sin^{-1}(20x) + \cos^{-1}(9x) + \tan^{-1}(2x)$ at $x = 0$.

$$\boxed{\begin{aligned} &y' = \frac{20}{\sqrt{1-400x^2}} - \frac{9}{\sqrt{1-81x^2}} + \frac{2}{1+4x^2} \\ &y'(0) = 20 - 9 + 2 = 13 \end{aligned}}$$

The answer is 13.

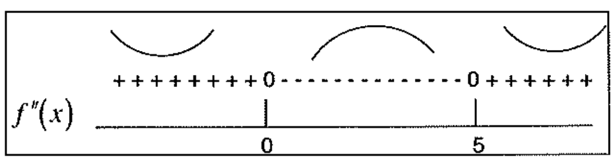
Location Problem: The temperature of a city for the 24-hour period starting at 12 noon is given by the equation $T(t) = 19 + 15\sin\left(\frac{\pi t}{12}\right)$ where t is the number of hours after 12 noon. Find the average temperature of the city to the nearest integer from 12 noon until 6 AM the next morning.

$$\boxed{\begin{aligned} &T_{ave} = \frac{\int_0^{18}\left[19 + 15\sin\left(\frac{\pi t}{12}\right)\right] dt}{18} \\ &T_{ave} = \frac{\left[19t - \frac{180}{\pi}\cos\left(\frac{\pi x}{12}\right)\right]_0^{18}}{18} = \frac{342 + \frac{180}{\pi}}{18} = 22.183 \end{aligned}}$$

The answer is 22.

Treasure Problem: The graph of $f(x) = \int_0^x \left(15t^2 - 2t^3 + 24\right) dt$ is concave up on (a,b). Find b - a.

$$\boxed{\begin{aligned} &f'(x) = \frac{d}{dx}\left[\int_0^x \left(15t^2 - 2t^3 + 24\right) dt\right] = 15x^2 - 2x^3 + 24 \\ &f''(x) = 30x - 6x^2 = 0 \\ &6x(5 - x) = 0 \Rightarrow x = 0, x = 5\, f(x) = \int_0^x \left(15t^2 - 2t^3 + 24\right) dt \end{aligned}}$$



The answer is 5.

 Stu Schwartz

AB Clue Problem Set # 18

Suspect Problem: Find the derivative of $y = \sin^{-1}(20x) + \cos^{-1}(9x) + \tan^{-1}(2x)$ at $x = 0$.

$$\frac{dy}{dx} = \frac{1}{\sqrt{1-(20x)^2}} \cdot 20 + \frac{-1}{\sqrt{1-(9x)^2}} \cdot 9 + \frac{1}{1+(2x)^2} \cdot 2$$

$$y'(0) = \frac{20}{\sqrt{1-400\cdot 0^2}} - \frac{9}{\sqrt{1-81\cdot 0^2}} + \frac{2}{1+4\cdot 0^2}$$

$$y'(0) = 20 - 9 + 2 = 13$$

The answer is: 13. Cross out that suspect number on your clue card and write # 18 as your set.

Location Problem: The temperature of a city for the 24-hour period starting at 12 noon is given by the equation $T(t) = 19 + 15\sin\left(\frac{\pi t}{12}\right)$ where t is the number of hours after 12 noon. Find the average temperature of the city to the nearest integer from 12 noon until 6 AM the next morning. $3\pi/2$

$u = \frac{\pi}{12}t$

$du = \frac{\pi}{12}dt$

$u(0) = 0$

$u(18) = \frac{18\pi}{12} = \frac{3\pi}{2}$

$$\frac{1}{18}\int_0^{18} 19 + 15\sin\left(\frac{\pi t}{12}\right) dt = \frac{1}{18}\left[\int_0^{18} 19\, dt + \frac{15\cdot 12}{\pi}\int_0^{3\pi/2} \sin u\, du\right]$$

$$= \frac{1}{18}\left[19t\Big|_0^{18} + \frac{180}{\pi}\left(-\cos u\Big|_0^{3\pi/2}\right)\right]$$

$$= \frac{1}{18}\left[342 + \frac{-180}{\pi}\left(\cos\frac{3\pi}{2} - \cos 0\right)\right]$$

$$= \frac{1}{18}\left[342 - \frac{180}{\pi}(0-1)\right] = \frac{1}{18}\left[342 + \frac{180}{\pi}\right] = 22.183$$

The answer is: 22. Cross out that location number on your clue card and write # 18 as your set.

Treasure Problem: The graph of $f(x) = \int_0^x \left(15t^2 - 2t^3 + 24\right) dt$ is concave up on (a,b). Find $b - a$.

f''

1. $f'(x) = \frac{d}{dx}\int_0^x (15t^2 - 2t^3 + 24)\, dt = 15x^2 - 2x^3 + 24$
2. $f''(x) = 30x - 6x^2$

   $0 = 6x(5-x)$

   $x = 0 \quad x = 5$

$f''(-1) \quad f''(1) \quad f''(6)$

$- \quad 0 \quad + \quad 5 \quad -$

$f(x)$ concave up from $(0, 5)$

$b - a = 5 - 0 = 5$

The answer is: 5. Cross out that treasure number on your clue card and write # 18 as your set.

AB Clue Problem Set # 19 Solutions

Suspect Problem: Let $f(x)=\begin{cases} ax^2+\frac{1}{3}, x\ge 1 \\ bx-\frac{10}{3}, x<1 \end{cases}$. If the function is differentiable, find the sum of $a+b$.

$$\lim_{x\to 1^+} f(x)=a+\frac{1}{3} \qquad \lim_{x\to 1^-} f(x)=b-\frac{10}{3}$$
$$a+\frac{1}{3}=b-\frac{10}{3}\Rightarrow b=a+\frac{11}{3}$$
$$f'(x)=\begin{cases} 2ax, x\ge 1 \\ b, x<1 \end{cases}$$
$$\lim_{x\to 1^+} f'(x)=2a \qquad \lim_{x\to 1^-} f'(x)=b$$
$$2a=b\Rightarrow 2a=a+\frac{11}{3}\Rightarrow a=\frac{11}{3} \text{ and } b=\frac{22}{3}$$
$$a+b=11$$

The answer is 11.

Location Problem: The rate of change of atmospheric pressure P with respect to the altitude h is proportional to P provided that the temperature is constant. At 15° C, the pressure is 101.3 pounds per square inch (psi) at sea level and 87.1 psi at height $h=1000$ m. Find the pressure in psi at the top of a mountain with an altitude of 8,200 meters. Round to the nearest integer.

$$\frac{dP}{dh}=kP\Rightarrow P=Ce^{kh}$$
$$101.3=Ce^0\Rightarrow C=101.3$$
$$P=101.3e^{kh}$$
$$87.1=101.3e^{1000k}$$
$$\frac{87.1}{101.3}=e^{1000k}$$
$$k=\ln\left(\frac{87.1}{101.3}\right)\Big/1000\approx -.000151$$
$$P=101.3e^{8300k}=28.92\approx 29$$

The answer is 29.

Treasure Problem: Find the area of the region bounded by the two functions $y=x^3$ and $y=3x-2$. Round to the nearest integer.

$$A=\int_{-2}^{1}\left[x^3-(3x-2)\right]dx \qquad x^3-3x+2=0\Rightarrow (x-1)^2(x+2)$$
$$A=\left[\frac{x^4}{4}-\frac{3x^2}{2}+2x\right]_{-2}^{1}$$
$$A=\frac{1}{4}-\frac{3}{2}+2-(4-6-4)=6.75$$

The answer is 7

AB Clue Problem Set # 19

Suspect Problem: Let $f(x)=\begin{cases} ax^2+\frac{1}{3}, x\geq 1 \\ bx-\frac{10}{3}, x<1 \end{cases}$. If the function is differentiable, find the sum of $a + b$.

• continuous when
$a(1^2)+\frac{1}{3}=b(1)-\frac{10}{3}$
$a+1/3=b-\frac{10}{3}$
$a+\frac{11}{3}=b$

• differentiable when
$2a(1)=b$
$2a=b$

• Find a & b
$a+\frac{11}{3}=2a$
$\frac{11}{3}=a$
$b=2a$
$b=2\left(\frac{11}{3}\right)=\frac{22}{3}$

• Sum of $a+b=\frac{11}{3}+\frac{22}{3}=\frac{33}{3}=\boxed{11}$

The answer is: 11. Cross out that suspect number on your clue card and write # 19 as your set.

$y=A_0e^{kt}$

Location Problem: The rate of change of atmospheric pressure P with respect to the altitude h is proportional to P provided that the temperature is constant. At 15° C, the pressure is 101.3 pounds per square inch (psi) at sea level and 87.1 psi at height $h = 1000$ m. Find the pressure in psi at the top of a mountain with an altitude of 8,200 meters. Round to the nearest integer.

(1) P: atmospheric pressure
h: altitude

Given constant temp: 15°C
$P=101.3$ at $h=0$ (sea level) $(0, 101.3)$
$P=87.1$ at $h=1000$ $(1000, 87.1)$

(2) find k
$y=A_0e^{kt} \rightarrow 87.1=101.3(e^{1000k})$ $A(1000)$
$\ln\left(\frac{87.1}{101.3}\right)=\cancel{\ln}e^{1000k}$
$\frac{\ln\frac{87.1}{101.3}}{1000}=k$
$-.000151\approx k$ Sto→A

(3) Find P when $h=8{,}200$ m
$A(8200)$
$y=101.3e$
$y\approx 29.360$

The answer is: 29. Cross out that location number on your clue card and write # 19 as your set.

Treasure Problem: Find the area of the region bounded by the two functions $y=x^3$ and $y=3x-2$. Round to the nearest integer.

(1) Find Limits
calc intersect or Long division
$x^3=3x-2$
$x^3-3x+2=0$
$(x+2)(x^2-2x+1)=0$
$(x+2)(x-1)(x-1)=0$
$x=-2$ $x=1$

(2) higher? check $x=0$
$y=x^3=0$ $y=3\cdot0-2=-2$

(1) Find Limits
$x+2\,\overline{)\,x^3+0x^2-3x+2}$ quotient x^2-2x+1
guess 6/c Last term ±2
$-(x^3+2x^2)$
$-2x^2-3x$
$-(-2x^2-4x)$
$x+2$
$-(x+2)$
0

(3) Area $\int_{-2}^{1} x^3-(3x-2)\,dx$
$=\frac{x^4}{4}-\frac{3x^2}{2}+2x\Big|_{-2}^{1}=$
$=\left[\frac{1}{4}-\frac{3}{2}+2\right]-\left[4-6-4\right]=\left[\frac{1}{4}-\frac{6}{4}+\frac{8}{4}\right]-\left[-6\right]$
$=\frac{3}{4}+6=6\frac{3}{4}\approx 7$

The answer is: 7. Cross out that treasure number on your clue card and write # 19 as your set.

 Stu Schwartz