7. AB Calculus – Step-by-Step **Name ________________________________**

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	$\frac{3\pi}{4}$	$\sqrt{2}$	0	$\frac{1}{a}$
2	8	-6	3	-4
3	1	5	4	$\frac{1}{2}$
4	5	$2a^2$	9	3

The functions f and g are differentiable for all real numbers x. The table above gives values of the function and their first derivatives at selected values of x with a being a constant.

a. If $h(x) = \sin(f(x))$, write an equation of the line tangent to h at the point where $x = 1$.

b. If $r(x) = \frac{1}{\sqrt{g(2x)}}$, find $r'(x)$ at $x = 2$.

c. Find the value(s) of a if the tangent lines to $f(g(x))$ and $g(f(x))$ are perpendicular at $x = 3$.

Parts a, b, and c all refer to $f(x)$, given by $f(x) = x^2 - x - 6$ which is defined on [0, 6].

a. Write an equation of the line tangent to f at the point where $x = 4$.

b. If $g(x) = [f(x)]^2$,write an equation of any horizontal tangent lines to g. Show how you arrive at your answer.

c. If $h(x) = \dfrac{1}{f(2x)}$, find all values of x where the tangent lines to h are either horizontal or do not exist on the interval [0, 6]. Show how you arrive at your answer.

9. AB Calculus – Step-by-Step **Name** ________________________________

Let $f(x)=\dfrac{e+\ln x}{x^2}$.

a. Find the average rate of change of f from $x = 1$ to $x = e$.

b. Write an equation of the line tangent to f at $x = 1$.

c. Find the x-coordinate of the point on f at which the tangent line to f is horizontal.

d. Find $\lim\limits_{x\to 0^+} f(x)$ and $\lim\limits_{x\to\infty} f(x)$.

10. AB Calculus – Step-by-Step **Name ________________________________**

Let $f(x)$ be given by the function $f(x)=\ln\left(x+\frac{1}{x}\right)$.

a. Show that $f'(x)=\frac{x^2-1}{x^3+x}$.

b. Find the x-coordinate of the point(s) on f at which the tangent line to f is horizontal.

c. Find the equation of the tangent line to $f(x)$ at $x=2$.

d. If $g(x)=e^{2f(x)}$, find $g'(e)$.

e. Show that $f(x)$ and $g(x)$ have horizontal tangent lines at the same x-value(s).

11. AB Calculus – Step-by-Step **Name** ________________________________

Consider the closed curve in the *xy*-plane given by $x^2 - 6x + y^3 - 12y = 11$.

a. Show that $\dfrac{dy}{dx} = \dfrac{6-2x}{3y^2-12}$.

b. Write an equation for the line tangent to the curve at the point (6, -1).

c. Find the coordinates of all points on the curve where the line tangent to the curve is vertical.

d. Show that it is impossible for this curve to have a horizontal tangent along the line $y = 4$.

12. AB Calculus – Step-by-Step (Calculator allowed) **Name** ________________________________

Consider the closed curve in the *xy*-plane given by $2x^2 - xy + y^3 + x = 9$.

a. Show that $\dfrac{dy}{dx} = \dfrac{y - 4x - 1}{3y^2 - x}$.

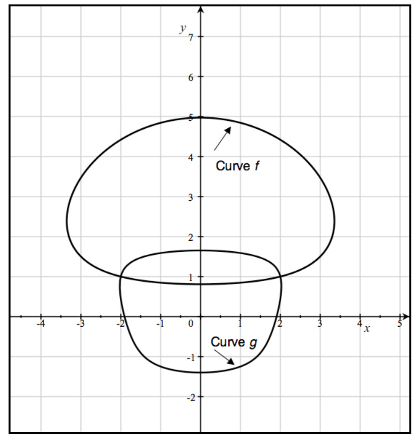
b. Find equation(s) of all tangent lines to the curve at $y = 1$.

c. There is a number k so that the point $(2.1, k)$ is on the curve. Using the tangent line found in part b, approximate the value of k.

d. Write an equation that can be solved to find the actual value of k so that the point $(2.1, k)$ is on the curve.

e. Solve the equation in part d) for k.

Curves f and g are given by the equations below as shown in the figure to the right.



Curve f: $x^2 - 9\ln(2y-1) + y^2 = 5$

Curve g: $x^2 + e^{y^2-1} - y = 4$

a. For curve f, show that $\dfrac{dy}{dx} = \dfrac{2xy - x}{9 - 2y^2 + y}$

b. Show that horizontal tangents to curve f must occur along the y-axis.

c. For curve g, find $\dfrac{dy}{dx}$.

d. Show that the line tangent to f is the same as the line normal to curve g at (2, 1).

14. AB Calculus – Step-by-Step **Name** ________________________________

Let $f(x)$ be given by the function $f(x)=\begin{cases} 9-4mx-(1-x)^2 & \text{if } x\le 1 \\ m^2x-n & \text{if } x>1 \end{cases}$ where m and n are constants and $m\ne 0$.

a. Write an expression for n if f is continuous at $x = 1$.

b. Show that f cannot be continuous at $x = 1$ if $n\le -14$.

c. If f is differentiable at $x = 1$, find the values of m and n. Show your reasoning.

d. Using the values of m and n found in part c), determine values of x (if any) that will make $f'(x)$ differentiable. Show your reasoning.

15. AB Calculus – Step-by-Step **Name** ________________________________

Let $f(x)$ be given by the function $f(x)=\begin{cases} g(x)+a & \text{if } x \le 0 \\ 3-b\cos x & \text{if } x > 0 \end{cases}$ where a and b are constants and $g(x)=\left|1-x^2\right|$.

a. Determine if $g(x)$ is differentiable at $x = 1$. Justify your answer.

b. Show that $f(x)$ is differentiable at $x = 0$ if $a = 1$ and $b = 1$.

c. Find a relationship between a and b in order for $f(x)$ to be continuous at $x = 0$.

d. Find a relationship between a and b in order for $f(x)$ to be differentiable at $x = 0$.