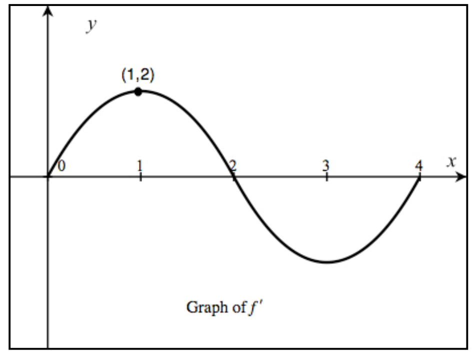
24. AB Calculus – Step-by-Step **Name** ________________________________

The figure to the right shows the graph of f', the derivative of the odd function f. This graph has horizontal tangents at $x = 1$ and $x = 3$. The domain of f is $-4 \le x \le 4$ and $f(1) = -3$.

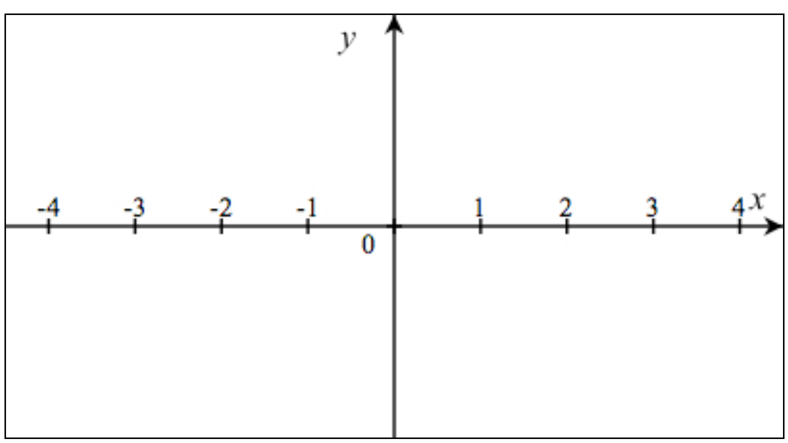


a. For what values of x does f on [-4,4] have a relative minimum and relative maximum? Justify your answers.

b. On what intervals is the graph of f concave upward? Justify your answers.

c. Find the equation of the tangent line to f at $x = -1$.

d. On the graph provided below, draw a sketch of the general shape of $f(x)$ for $-4 \le x \le 4$ which passes through the origin.



25. AB Calculus – Step-by-Step **Name** ________________________________

Consider a differentiable function f having domain all positive real numbers and $f(x)=\dfrac{3x^2-2}{3x^3}$.

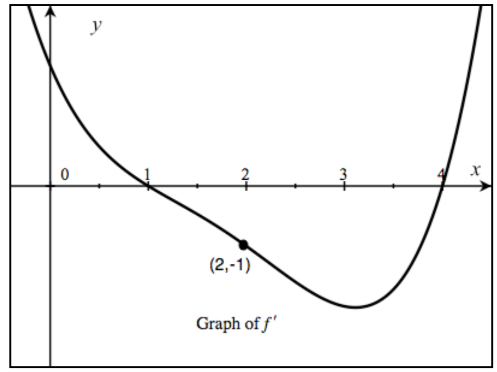
a. Show that $f'(x)=\dfrac{2-x^2}{x^4}$.

b. Find the x-coordinate of the critical point of f. Determine whether the point is a relative maximum, relative minimum, or neither. Justify your answer.

c. Find intervals where the graph of f is concave up. Justify your answer.

d. Find the value of x where the tangent line to $f(x)$ is parallel to the line $y=x$. Explain your reasoning.

Let f be a twice-differentiable function defined on the interval $-0.5 < x < 4.5$ with $f(2) = -3$. The graph of f', the derivative of f is shown to the right. The graph of f' has x-intercepts at $x = 1$ and $x = 4$ and has a horizontal tangent at $x = 3$. Let g be the function given by $g(x) = e^{-f(x)}$.



a. Write an equation for the line tangent to the graph of g at $x = 2$.

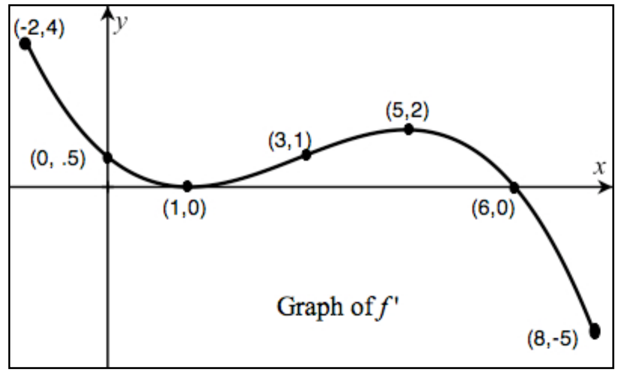
b. For $-0.5 < x < 4.5$, find all values of x at which g has a local maximum. Justify your answer.

c. Find the average rate of change of g', the derivative of g, on the interval [2, 4].

d. The second derivative of g is given by $g''(x) = -e^{-f(x)}\left[f''(x) - (f'(x))^2\right]$. Determine whether g is concave up or concave down at $x = 1$. Justify your answer.

27. AB Calculus – Step-by-Step **Name** ________________________________

The figure to the right shows the graph of f', the derivative of the function f on the closed interval $-2 \le x \le 8$. The graph of f' has horizontal tangents at $x = 1$ and $x = 5$. The function is twice differentiable with $f(3) = -2$.



a. Find the x-coordinate of the point(s) of inflection of the graph of f. Give a reason for your answer.

b. For what values of x does f attain its absolute maximum value on the closed interval $-2 \le x \le 8$? Show the analysis that leads to your answer.

c. Using the known points given on the graph of f', for what value(s) of x does the graph of $y = x^2 + f(x)$ have a horizontal tangent? Give a reason for your answer.

d. Let g be the function defined as $g(x) = x^2 f(x)$. Find an equation for the line tangent to the graph of g at $x = 3$.

A particle moves along the x-axis so that any time $t > 0$, its velocity is given by $v(t) = 2t\ln t - t$.

a. Write an expression for the acceleration of the particle.

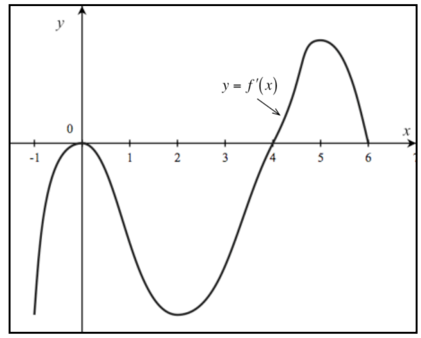
b. What are the values of t for which the particle is moving to the right? Justify your answer.

c. Is the particle speeding up or slowing down at $t = 1$? Show the analysis that leads to your conclusion.

d. Find the absolute minimum velocity of the particle. Show the analysis that leads to your conclusion.

29. AB Calculus – Step-by-Step **Name** ______________________________

Let f be a function that has domain: the closed interval [-1, 6] and range: the closed interval [-10, 2]. Let $f(-1)=2, f(0)=0$, and $f(6)=-2$. Let f have the derivative f' that is continuous and have the graph shown in the figure above.



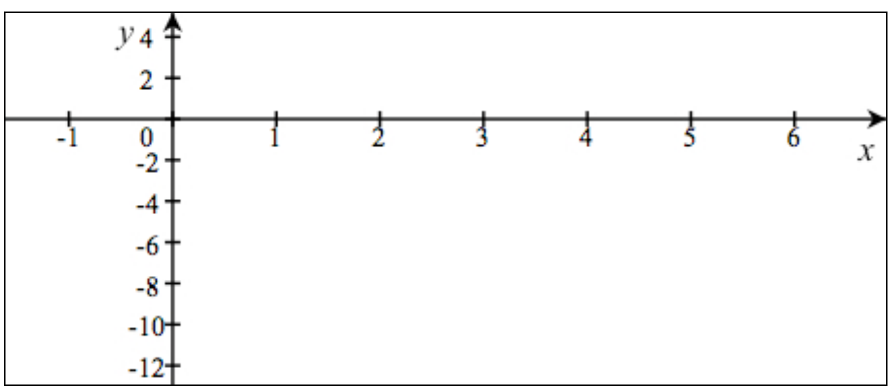
a. Find all values of x for which f assumes a relative minimum. Give a reason for your answer.

b. Find all values of x for which f assumes its absolute maximum. Justify your answer.

c. Find the intervals on which f is concave upward.

d. Find all values of x for which f has a point of inflection. Give a reason for your answer.

e. On the axes provided, sketch the graph of f.



30. AB Calculus – Step-by-Step **Name ________________________________**

In the Angry Birds™ game, the green bird (also called the Boomerang Bird) can change directions. Suppose the green bird is catapulted along the x-axis such that its position at time t is given by $x(t) = 4\cos(\pi t^2) - 1$ for $0 \le t \le \sqrt{\frac{3}{2}}$.

a. Find an expression for the velocity of the bird.

b. For what values of t is the bird moving left? Justify your answer.

c. Is the bird slowing down, speeding up, or neither at $t = \frac{\sqrt{3}}{2}$? Show the analysis that leads to your conclusion.

d. How far does the bird travel by the time it reaches its absolute minimum value on the x-axis?

31. AB Calculus – Step-by-Step **Name** ________________________________

Suppose f is a function defined on [-8, 8] given by $f(x) = 4x^{1/3} - x^{4/3} - k$, where k is a positive constant.

a. Show that $f'(x) = \dfrac{4-4x}{3x^{2/3}}$.

b. For what values of x is $f(x)$ increasing? Justify your answer.

c. Write an expression for the absolute minimum value of f on [-8, 8]. Show the analysis that leads to your answer.

d. Find all possible values of k such that $f(x)$ has no real zeros. Show the analysis that leads to your answer.

32. AB Calculus – Step-by-Step **Name** ________________________________

Suppose f is a function given by $f(x)=\left(x^2-2x-14\right)e^{-x}$.

a. Find the interval(s) where f is increasing. Justify your answer.

b. Find the x-value where there are point(s) of inflection for $f(x)$.

c. Find the absolute maximum and absolute minimum values of f if they exist. Show the analysis that leads to your conclusion.

33. AB Calculus – Step-by-Step (Calculators Allowed) **Name ________________________________**

Consider the function $f(x)=\ln(x+1)-\sin x$ defined on $0\le x\le 2\pi$.

a. Find the equation of the tangent line to f at $x=\pi$.

b. Find the minimum slope of $f(x)$ for $0\le x\le 2\pi$. Show the analysis that leads to your conclusion.

c. If the function $g(x)=\ln(x+1)-k\sin x$ has a critical point at $x=\pi$, find the value of k and determine whether the point $(\pi, g(\pi))$ is a relative minimum, relative maximum, or neither for $g(x)$. Show the analysis that leads to your conclusion.

34. AB Calculus – Step-by-Step **Name** ________________________________

Let f be a function defined for all $x \neq 0$ such that $f(5) = 2$ and the derivative of f is given by $f'(x) = \dfrac{x^2 - 10x + 16}{x}$ for all $x \neq 0$.

a. Find all values of x for which the graph has a relative maximum and relative minimum. Justify your answer.

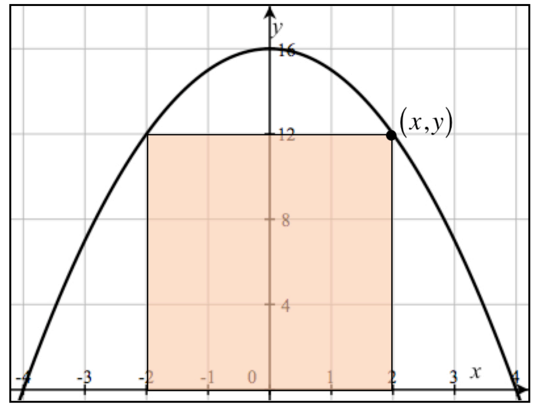
b. Find the minimum slope of f if $x > 0$.

c. Find the equation for the line tangent to the graph of f at $x = 5$ and use it to approximate $f(5.5)$.

d. Does the value found in part c) underestimate or overestimate $f(5.5)$? Give a reason for your answer.

35. AB Calculus – Step-by-Step Name ________________________________

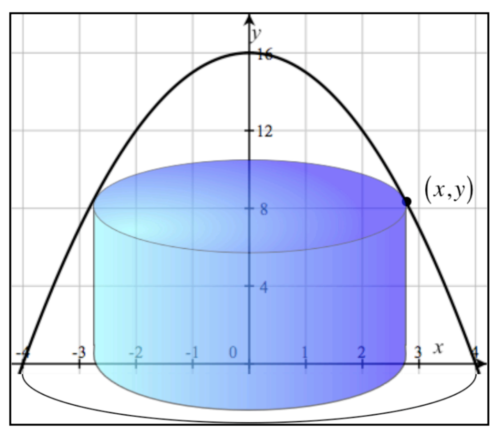
A rectangle is inscribed in the region bounded by the x-axis and the parabola $y = 16 - x^2$ as shown in the figure to the right.



a. The point shown in the figure moves along the curve so that its x-coordinate increases at the constant rate of 1.5 units/minute. Find the rate of change of the area of the rectangle when $x = 2$.

b. Find the dimensions of the rectangle that gives the greatest area.

c. The parabola $y = 16 - x^2$ is rotated about the y-axis to form a paraboloid. A cylinder is inscribed in the paraboloid as shown in the figure to the right. Find the radius and height of the cylinder of greatest volume.



36. AB Calculus – Step-by-Step (Calculators Allowed) **Name ________________________________**

The price of a share of stock in dollars over a week is given by the function $P(t) = \sqrt{2t+1} + 2\cos t + 20$ where t is measured in days and $0 \le t \le 5$.

a. Find the average rate of change of the price of the stock over [0, 5]. Use correct units.

b. Apply the Mean-Value Theorem to P on [0, 5] and explain the result in the context of the problem situation.

c. On what value of t over the 5-day period is the price of the stock increasing the fastest?