37. AB Calculus – Step-by-Step **Name** ________________________________

Let f and g be functions whose derivatives are given by

$$f'(x) = 3x^2 - 2x - 2 \quad \text{and} \quad g'(x) = \frac{x-1}{\sqrt{x}}$$

a. If it is known that $f(1) = 5$, find $f(x)$.

b. Using the answer from part a, find the x-value(s) guaranteed by the Mean-Value Theorem on f on the interval (-1, 1)

c. Show that the answer to part b) would be the same if there was no information given about the value of f at any x-value.

d. Find the average rate of change of g on [1, 4].

38. AB Calculus – Step-by-Step **Name** ________________________________

Let $f'(x) = \dfrac{1}{\sqrt{2x+1}}$ and $f(0) = 2$.

a. Find $f(x)$.

b. Find the value of c guaranteed by the Mean-Value Theorem on [0, 12].

c. Find the value of k, $k > 0$, such that the average rate of change of f on $[0, k]$ is $\dfrac{1}{2}$.

39. AB Calculus – Step-by-Step **Name** ________________________________

Let $F'(x)=\begin{cases} f'(x) & \text{if } x \ge 2 \\ g'(x) & \text{if } x < 2 \end{cases}$ where $f'(x)=\dfrac{30x^2}{\left(x^3-3\right)^2}$ and $g'(x)=x\cos\left(\pi x^2\right)$.

a. If $f(2)=4$, find $f(x)$.

b. If $F(x)$ is continuous at $x=2$, find $g(x)$.

c. If $F(x)$ is continuous at $x=2$, determine if $F(x)$ is differentiable at $x=2$. Show the analysis that leads to your conclusion.

40. AB Calculus – Step-by-Step **Name** ______________________________

A particle is moving along the x-axis with velocity $v(t) = e^t \cos(e^t - 1) = 0$. At time $t = 0$, the particle is at $x = 2$.

a. For what value of t, $t > 0$, is the particle first stopped?

b. Find the position function $x(t)$.

c. How far does the particle travel from $t = 0$ to the time it is first stopped?

d. What is the acceleration of the particle at the time it is first stopped?

An elevator is being tested and travels from floor to floor without ever opening its door to pick up people. During the time interval $0 \le t \le 60$ seconds, the elevator's velocity v, measured in feet/sec, and acceleration a, measured in feet per second per second are continuous functions. The table below shows values of these functions at selected times.

t (sec)	0	5	10	20	25	35	45	55	60
$v(t)$ ft/sec	-10	-10	-5	0	10	15	5	0	-5
$a(t)$ ft/sec^2	2	3	3	2	1	0	-1	-3	-2

a. Approximate $\int_0^{60} |v(t)|\, dt$ using a left Riemann sum with eight subintervals and explain its meaning in terms of the elevator's motion using appropriate units.

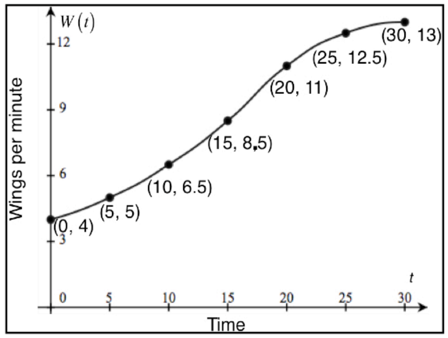
b. Approximate $\int_{10}^{45} a(t)\, dt$ using a right Riemann sum with four subintervals using appropriate units.

c. Approximate $\frac{1}{35}\int_{20}^{55} v(t)\, dt$ using a midpoint approximation with two subintervals and explain its meaning in terms of the elevator's motion using appropriate units.

d. Based on the table, for what values of t is the elevator speeding up? Why?

42. AB Calculus – Step-by-Step **Name** ________________________________

Wing Bowl is an annual eating contest founded by the Morning Show on Philadelphia's WIP Radio as a celebration of gluttony. The event that attracts more than 20,000 people is a contest to see who can eat the most chicken wings in 30 minutes.



The rate of wing consumption, in wings per minute, recorded during a past Wing Bowl of the champion El Wingador, is given by a twice-differentiable and strictly increasing function W of time t. The graph of W for the time interval $0 \le t \le 30$ minutes is shown.

a. Use the data from the table to find an approximation for $W'(18)$. Show the computations that lead to your answer. Indicate units of measure.

b. The rate of wing consumption for El Wingador is increasing fastest at time $t = 18$ minutes. What is the value of $W''(18)$? Explain your reasoning.

c. Approximate the value of $\int_0^{30} W(t)\,dt$ using a left Riemann sum with the six subintervals indicated by the data in the table. Is this numerical approximation less than the value of $\int_0^{30} W(t)\,dt$? Explain.

d. Explain the meaning of $\int_0^{30} W(t)\,dt$ and $\dfrac{\int_0^{30} W(t)\,dt}{m}$ where m =30 minutes in terms of wing consumption for El Wingador. Indicate units of measure for both answers.

43. AB Calculus – Step-by-Step **Name** ________________________________

Due to a bad storm on a low-lying road, a large circular puddle of water forms. The area of the puddle increases as the storm intensifies. The radius of the puddle, in feet, is modeled by a twice-differentiable function r of time t, where t is measured in minutes. For $0 < t < 15$, the graph of r is concave up. The table below gives selected values of the rate of change, $r'(t)$ of the radius of the puddle over the time interval $0 \le t \le 15$. The radius of the puddle is 7 feet when $t = 6$.

t (minutes)	0	3	6	8	11	12	15
$r'(t)$ (feet per minute)	1.2	2.3	3.4	4.3	4.9	5.0	6.2

a. Estimate the radius of the puddle when $t = 5$ using the tangent line approximation at $t = 6$. Is your estimate greater than or less than the true value? Give a reason for your answer.

b. Find the rate of change of the area of the puddle with respect to time at $t = 6$. Indicate units of measure.

c. Use a right Riemann sum with six intervals using the data in the table to approximate $\int_0^{15} r'(t)\,dt$. Using correct units, explain the meaning of $\int_0^{15} r'(t)\,dt$ in the context of the problem situation.

d. Is your approximation in part c) greater or less than $\int_0^{15} r'(t)\,dt$? Give a reason for your answer.

44. AB Calculus – Step-by-Step (Calculators Allowed) **Name** ______________________________

t (minutes)	0	5	10	15	20	25	30
$P(t)$ (people)	0	35	85	125	185	220	275

The Staten Island Ferry has people waiting in a waiting room for up to 30 minutes. At the end of that time, the doors open and people board the ferry. The boarding process takes 5 minutes. The number of people in the waiting room is modeled by a differentiable function P for $0 \le t \le 30$. Values of $P(t)$ are shown in the table above at 5-minute intervals.

a. Use the data in the table to approximate the rate of change, in people per minute, of people in the waiting at $t = 16$ minutes.

b. Use a trapezoidal sum with six intervals given by the table to approximate the value of $\frac{1}{30}\int_0^{30} P(t)\,dt$.

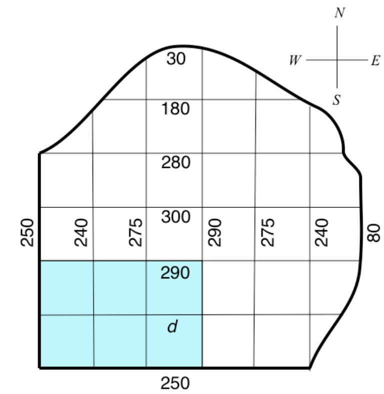
Using correct units, explain the meaning of $\frac{1}{30}\int_0^{30} P(t)\,dt$ in terms of people in waiting room.

c. Once the doors to the waiting room close, the doors to the ferry open and stay open for 5 minutes. People board the ferry at the rate modeled by the function $B(t) = t^3 - 15t^2 + 60t + 10$ for $0 \le t \le 5$. What is the difference in boarding rate from the time the boarding door opens until the time it closes? Specify units.

d. Using $B(t)$ from part c) at what time are people boarding the ferry the fastest? Justify your answer.

45. AB Calculus – Step-by-Step (Calculators Allowed) **Name** ________________________________

A pond that is iced over is situated at the intersection of two perpendicular roads as shown in the figure to the right. George and Jerry want to estimate the area of the pond. George does the north-south measurements while Jerry does the east-west measurements, both in feet. Each north-south line and east-west line is separated by 50 feet. Because Elaine was distracting him, there is one measurement that Jerry failed to make and is labeled as d.



a. Both George and Jerry do their area calculations using three midpoint Riemann sums. If Jerry's estimation is 500 feet^2 less than George's estimation, what is the value of d? Show the computations that lead to your answer.

b. Both George and Jerry do their area estimations using trapezoids. Jerry uses six trapezoids while George uses three equally spaced trapezoids. If Jerry's estimation is 4% more than George's estimation, what is the value of d ? Show the computations that lead to your answer.

c. The shaded region of the pond is used for kids to ice skate. Not knowing that, in that section, Kramer cuts out a circular hole 30π feet in circumference to ice-fish. What area of the shaded region is actually ice?

d. When the radius of the hole is 30π feet in circumference, the radius of the hole is increasing at the rate of 2 feet/day. Find the rate of change of the percentage of the shaded region that is actually ice.