46. AB Calculus – Step-by-Step **Name** ________________________________

Let f be a continuous function that is defined for all real numbers x and has the following properties.

(i) $\int_2^5 f(x)\,dx = \frac{-3}{4}$ (ii) $\int_2^8 f(x)\,dx = 3$

a. Find $\int_8^5 f(x)\,dx$

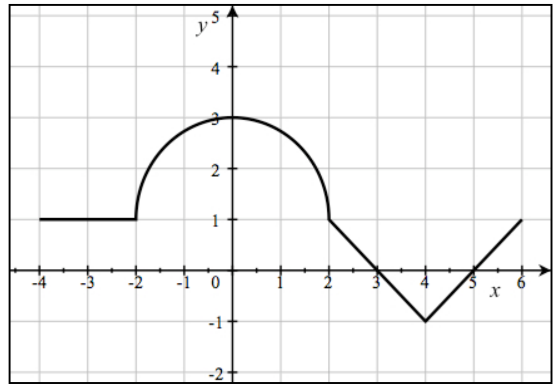
b. Find $\int_0^3 \left[2f(x+2) - x^2\right] dx$

c. Given that $f(x) = ax + b$, find a and b.

d. Using the values of a and b from c), find the value of k, $k \neq 0$, such that $\int_0^k f(x)\,dx = 0$.

47. AB Calculus – Step-by-Step **Name** ________________________________

The graph of a function f consists of a semicircle and three line segments as shown above. Let g be the function given by $g(x) = \int_0^x f(t)\,dt$.



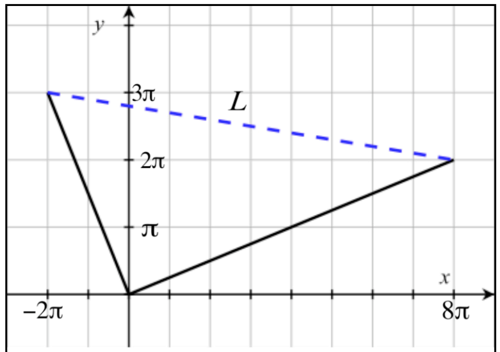
a. Find $g(3)$.

b. Find the absolute minimum value of g on (-4, 6). Justify your answer.

c. Find the equation of the tangent line to g at $x = 2$.

d. Find the x-coordinate of each point of inflection of the graph of g on (-4, 6). Justify your answer.

Let f be the piecewise-linear function defined on $[-2\pi, 8\pi]$ whose graph is shown to the right (two lines passing through the origin) and let $g(x) = f(x) + \sin\left(\dfrac{x}{4}\right)$.



Graph of f

a. Find $\displaystyle\int_{-2\pi}^{8\pi} g(x)\,dx$.

b. Find all x-values in the interval (-2π, 8π) for which g has a critical value.

c. Suppose $h(x)$ is defined by line L as shown in the figure above. Find $\displaystyle\int_{-\pi}^{\pi} h(x)\,dx$.

49. AB Calculus – Step-by-Step (Calculators Allowed) **Name** ________________________________

A cruise ship leaves a port and travels at the speed given by $r(t) = 18\left(1 - e^{-8t^2}\right)$, $t \geq 0$, measured in knots (nautical miles per hour). The amount of fuel in gallons used by the ship to travel x miles is modeled by $f(x) = 80x\left(1 - e^{-x}\right)$.

a. The maximum speed of the cruise ship is $\lim\limits_{t \to \infty} r(t)$. What is the maximum speed of the ship in knots?

b. How many nautical miles does the ship travel in the first two hours?

c. Find the rate of change with respect to time of the number of gallons of fuel used by the ship when t = 2 hours. Indicate units of measure.

d. How many gallons of fuel have been used by the ship when it reaches a speed of 10 knots?

50. AB Calculus – Step-by-Step (Calculators Allowed) **Name ________________________________**

Build Your Own Burger (BYOB) allows people to decide exactly how large they want their burger. Burgers sell for \$1.50 per ounce. The restaurant's cost to actually make the burger varies with its size. BYOB states that if x is the size of the burger in ounces, for each ounce the cost is $\frac{1}{2}x^{1/3}$ dollars per ounce.

a. Use definite integrals to express and find the profit on the sale of an 8-ounce burger.

b. Using correct units, explain the meaning of $10\int_{8}^{12}\frac{1}{2}x^{1/3}dx$ in the context of this problem. Do not calculate.

c. If k is the size of a burger in ounces that someone ordered, write an expression for $P(k)$, the amount of profit that BYOB makes on that burger.

d. It is possible for BYOB to actually lose money if someone purchases a really huge burger because of the need to create a large enough bun, the cost of condiments, etc. Find the maximum profit that BYOB could earn on the purchase of one burger. Justify your answer.

51. AB Calculus – Step-by-Step (Calculators Allowed) **Name ________________________________**

The rate that people enter a Broadway theatre is modeled by the function R given by $R(t)=\frac{t^2}{2}-\frac{t^3}{60}$, for $0 \le t \le 30$ minutes. $R(t)$ is measured in people per hour. There are no people in the theatre at $t = 0$ when the doors open and the show begins at $t = 30$.

a. How many people are in the theatre when the show begins?

b. Find the time when the rate at which people enter the theatre is increasing the fastest. Express this rate using the proper units. Justify your answer.

c. The total wait time for all the people in the theatre is found by adding the time each person waits for the show to begin, starting at the time the person enters the theatre. The function w models the total wait time for all the people who enter the theatre before time t. The derivative of w is given by $w'(t)=(30-t)R(t)$. Find $w(30)-w(0)$, the total wait time for people who enter the theatre after time $t = 0$.

d. On average, how long does each person who sees the show wait in the theatre for the show to begin? Use the wait time model from part c).

52. AB Calculus – Step-by-Step **Name** ________________________________

Let $F(x) = \int_0^x \cos(\pi t^2)\, dt$ for $0 \le t \le 2$.

a. Use the trapezoidal rule with four equal subdivisions on the closed interval [0, 2] to approximate $F(2)$.

b. For what values of x does F have relative maximums? Justify your answer.

c. If the average rate of change of F on the closed interval [0, 2] is given by $2k-1$, solve for k in terms of $\int_0^2 \cos(\pi t^2)\, dt$.

 Name ________________________________

Let $f(x)=\int_0^{2x}\sqrt{1+t^2}\,dt$ and $g(x)=f(2\cos x)$

a. Find $f'(x)$ and $g'(x)$.

b. Write an equation for the line tangent to the graph of $y=g(x)$ at $x=\frac{\pi}{2}$.

c. Write, but do not evaluate, an integral expression that represents the absolute minimum value of g on the interval $\frac{\pi}{2}\le x\le\frac{3\pi}{2}$. Justify your answer.

54. AB Calculus – Step-by-Step (Calculators Allowed) **Name** ________________________________

An area was hit by a weekend snowstorm. On Monday morning, the temperature rises allowing the snow to melt. At 12 AM, Monday morning, the height of the snow is 8 inches. The height of the snow changes at a rate modeled by $f(t) = \sin(t) - \sqrt{5-t}$, measured in inches/day, t days after 12 AM Monday.

a. What is the height of the snow at 12 AM on Wednesday morning?

b. If $F(t) = \int_0^t f(x)\,dx$, using correct units, find and interpret the value of $F'(2)$ in terms of the height of the snow.

c. At what time during the 5-day week was the snow level falling the fastest? Justify your answer.

d. Write, but do not solve, an equation involving an integral expression whose solution k would be the number of days the height of the snow would be half of its height at 12 AM Monday.

55. AB Calculus – Step-by-Step **(Calculators Allowed)** **Name** ________________________________

The Assembly Line Restaurant serves pizza with a twist. There is a conveyor belt that starts in the kitchen and runs the length of the restaurant. Customers sit on both sides of the belt. During one hour, there is an all-you-can-eat buffet. Pizza slices are put on the belt in the kitchen and customers take them off and eat them in the dining room. Uneaten slices are placed in a container at the end of the belt, and customers can eat them as well.

Slices are placed on the belt at a rate modeled by the function B, given by $B(t)=\dfrac{60t}{9+4t}$.

Slices are eaten at a rate modeled by the function E, given by $E(t)=12+4\left(1+\dfrac{t}{40}\right)\sin\left(\dfrac{2\pi t}{45}\right)$.

Both $B(t)$ and $E(t)$ have units of slices per minute and t is measured in minutes for $0 \le t \le 60$. At time $t = 0$, there are 100 slices in the container.

a) Write and evaluate an expression for how many slices are eaten during the hour period.

b) Write an expression for $S(t)$, the number of slices on the belt or in the container at time t.

c) Find the rate at which the number of slices on the belt or in the container is changing at time $t = 10$.

d) For $0 \le t \le 60$, at what time is the number of slices on the belt or container a minimum? What is the minimum value? Justify your answer.