56. AB Calculus – Step-by-Step **Name** ______________________________

Relative humidity is a term used to describe the amount of water vapor in a mixture of air and water vapor. A storm is passing through an area and the relative humidity changes rapidly. The relative humidity in percentage is a differentiable function H of time t. The table below shows the relative humidity as recorded over a 12-hour period.

t (hours)	0	2	4	6	8	10	12
$H(t)$ (percent)	40	89	94	70	50	43	41

a. Use data from the table to find an approximation for $H'(6)$. Show the computations that lead to your answer. Indicate units of measure.

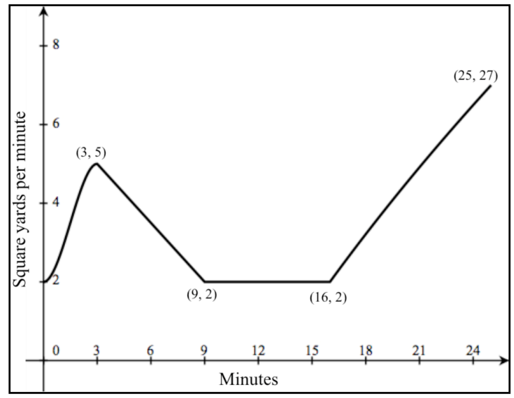
b. Approximate the average relative humidity in percent over the time period $0 \le t \le 12$ using a trapezoidal approximation with subintervals of length $\Delta t = 2$ hours.

c. A meteorologist proposes the function R, given by $R(t) = 40 + 30te^{-t^2/20}$, as a model for the relative humidity in the air at time t, where t is measured in hours and $R(t)$ is measured in percent. Find $R'(6)$. Using appropriate units, explain the meaning of your answer.

d. Use the function R defined in part c) to find the average relative humidity on the time interval $0 \le t \le 12$ hours.

57. AB Calculus – Step-by-Step (Calculators Allowed) **Name** ________________________________

Tim is mowing a lawn with an uneven terrain forcing him to push the mower at a rate modeled by the piecewise function f. In the figure to the right, $f(t)=\dfrac{-2}{9}t^3+t^2+2$ for $0\le t\le 3$ and f is piecewise linear for $3<t\le 16$. For $16<t\le 25$, f is modeled by $f(t)=5\sqrt{t}-18$. The function f is measured in $\dfrac{\text{yd}^2}{\text{min}}$ and t is measured in minutes.



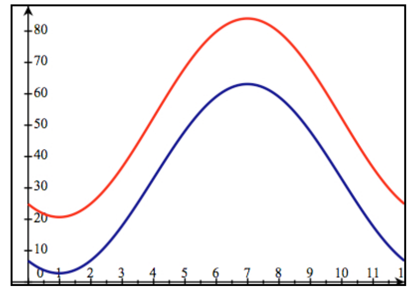
a. Find $f'(5)$. Indicate units of measure.

b. For the time interval $0\le t\le 25$, at what time t is f increasing at the greatest rate? Show the reasoning that supports your answer.

c. Find the average number of square yards Tim cuts per minute on $3\le t\le 16$.

d. Tim adjusts his rate of cutting so that he cuts $\left(f(t)+k\right)\ \dfrac{\text{yd}^2}{\text{min}}$ on $3\le t\le 16$. Find the value of k if Tim wants to average $4\ \dfrac{\text{yd}^2}{\text{min}}$ for $3\le t\le 16$.

58. AB Calculus – Step-by-Step (Calculators Allowed) **Name** ________________________________



Because of its location in the center of North America, the Twin Cities region in Minnesota is subjected to some of the widest range of temperatures in the United States. The high temperatures over a year is modeled by the equation $T_H = 31.44\sin\left(\frac{\pi x}{6} - 2.09\right) + 52.56$ while the low temperatures over a year is modeled by the equation $T_L = 30.15\sin\left(\frac{\pi x}{6} - 2.09\right) + 32.95$. The graph of these functions is shown in the figure on the right. The variable x refers to the 15th day of the month where 0 represents December, 1 represents January, … and 12 represents November and T is measured in degrees Fahrenheit.

a. What is the average rate of change of the high temperature from January 15 to July 15? Specify units.

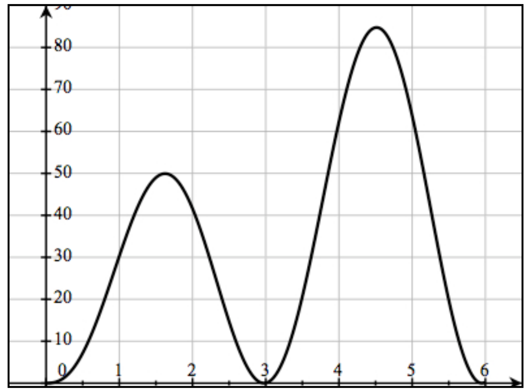
b. What is the average difference between the high and low temperatures over a year? Specify units.

c. During what month is the low temperature rising the fastest? Justify your answer.

d. The Newton family has programmed their air conditioner to run whenever the average high temperature is over 72 degrees. The cost of cooling the house accumulates at the rate of \$40 per month for each degree the average high temperature is over 72 degrees. What is the total cost to the nearest dollar for cooling the house over a year?

59. AB Calculus – Step-by-Step (Calculators Allowed) **Name** ______________________________

A popular Comedy Club has two shows, the early show from 6 PM until 9 PM and the late show from 9 PM until 12 midnight. The rate that people enter the club is given by $R(t) = 40\sqrt{t}\sin^2\left(\frac{19}{18}t\right)$ people per hour over the time interval $0 \le t \le 6$ where $t = 0$ corresponds to 6 PM. When the early show is over at 9 PM, the club clears and people come in for the late show. The graph of $R(t)$ is to the right.



a. Find the difference between the number of people who come to the early show and those who come to the late show. Round to the nearest person.

b. The club hires an extra maitre d' when more than 60 people per hour are entering the club. Find the average number of people in the club during that period of time.

c. The club brings in extra waiters when the number of people in the club is greater than 75. Write, but do not solve, an equation using an integral that solves for time t when extra waiters are first needed.

d. The club cuts off all alcoholic drinks one hour before the late show ends. If the average early show customer spends \$15 on drinks and the average late show customer spends \$25 on drinks, what is the average amount of money spent on drinks per hour during the evening?

60. AB Calculus – Step-by-Step **Name** ________________________________

A particle moves along the y-axis with velocity at time $t \geq 0$ given by $v(t) = e^{2t-4} - 1$.

a. Find the acceleration of the particle at time $t = 1$.

b. Is the particle speeding up or slowing down at $t = 1$? Give a reason for your answer.

c. Find all values of t for which the particle changes direction. Justify your answer.

d. Find the total distance traveled by the particle over $0 \leq t \leq 4$.

61. AB Calculus – Step-by-Step **Name** ________________________________

Roller coaster A leaves the station at time $t = 0$. The velocity of the coaster is recorded for selected values of t over the interval $0 \le t \le 12$ seconds, as shown in the table below.

t (seconds)	0	2	4	6	8	10	12
$v(t)$ (feet per second)	8	12	22	35	55	76	80

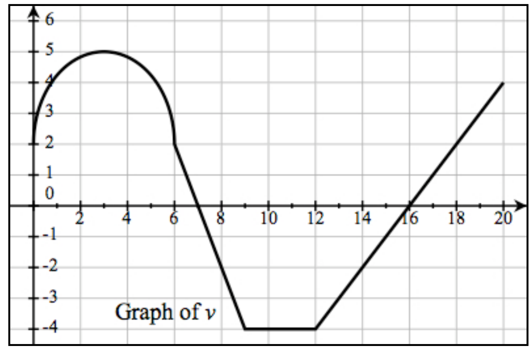
a. Find the average acceleration of roller coaster A over the time interval $0 \le t \le 12$. Indicate units of measure.

b. Using correct units, explain the meaning of $\frac{1}{12}\int_0^{12} v(t)\,dt$ in terms of the roller coaster's travel. Use a midpoint Riemann sum with 3 subintervals of equal length to approximate $\frac{1}{12}\int_0^{12} v(t)\,dt$.

c. Roller coaster B travels on a parallel track to coaster A. Its acceleration is given by $a(t) = \frac{12}{\sqrt{4t+1}}$ feet per second per second and its initial velocity is 40 feet per second. Which of the two coasters is traveling faster at $t = 12$ seconds? Explain your answer.

62. AB Calculus – Step-by-Step **Name** ________________________________

A searchlight is trained on the intersection of a building and a wall. At $t = 0$, the light travels along the wall in a straight line. For $0 \le t \le 20$, the velocity of the light is modeled by the piecewise function defined by the graph to the right. The graph is made of lines and a semi-circle.



a. At what times in the interval $0 < t < 20$, if any, does the light change direction? Give a reason for your answer.

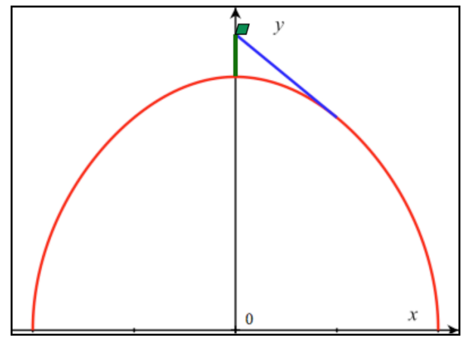
b. At what times in the interval $0 \le t \le 20$ is the light farthest from the building? How far is the light from the building at this time?

c. Find the total distance the light travels during the time interval $0 \le t \le 20$.

d. Write expressions for the light's acceleration $a(t)$, velocity $v(t)$, and distance $x(t)$ from the building that are valid to the time interval $6 < t < 9$.

63. AB Calculus – Step-by-Step (Calculators Allowed) **Name** ______________________________

The cross-section of a hill is modeled by $y = 60\left(\dfrac{\pi}{2} - \sin^{-1}\left(x^2\right)\right)$, measured in feet as shown in the figure to the right.



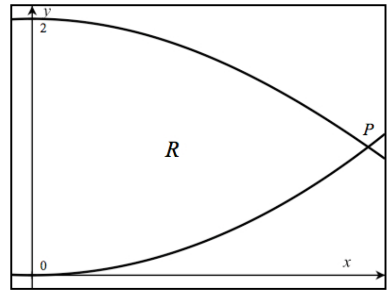
a. Find $\dfrac{dy}{dx}$.

b. At the very top of the hill, there is a flagpole. The pole is supported by a wire that is tangent to y at $x = \dfrac{1}{2}$ and extends to the very top of the pole. Find the equation of the line that represents the wire.

c. Find the height in feet of the flagpole.

d. Find the average height of the hill.

 Name ________________________________



Let R be the region in the first quadrant enclosed by the graphs of $f(x) = 2\cos \pi x$ and $g(x) = 9x^2$ which intersect at point P, which has x-coordinate of $\frac{1}{3}$, as shown in the figure to the right.

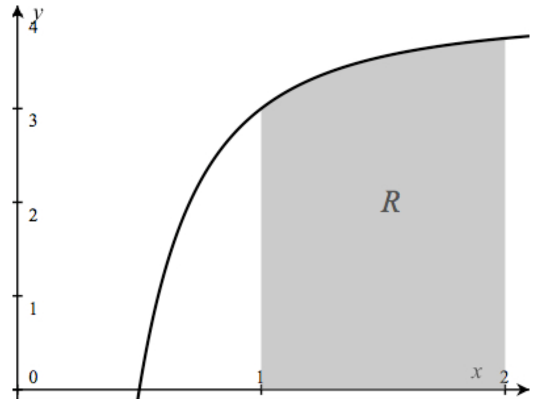
a. Find the equation of the tangent line to the graph of f at point P.

b. Find the area of R.

c. Write, but do not evaluate, an integral expression for the volume of the solid generated when R is rotated about the horizontal line $y = 2$.

65. AB Calculus – Step-by-Step **Name** ________________________________

Let R be the region in the first quadrant enclosed by the graph of $f(x) = 4 - \frac{1}{x^2}$, the x-axis and the lines $x = 1$ and $x = 2$, as shown in the figure to the right.



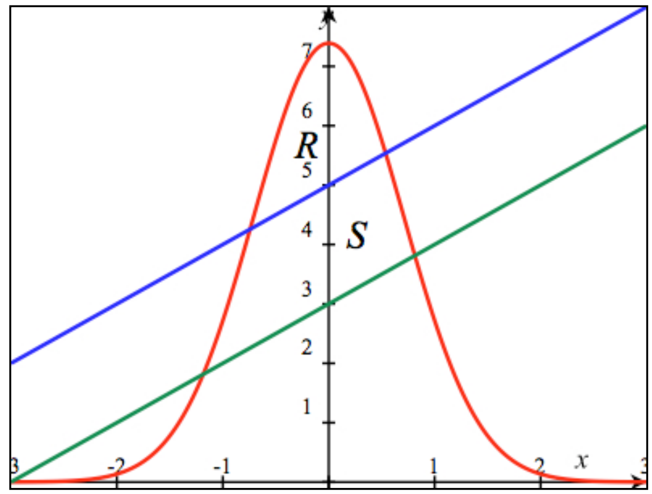
a. Find the area of R.

b. Write, but do not evaluate, a simplified integral expression that gives the volume of the solid generated when R is rotated about the line $y = 4$.

c. Region R is the base of a solid. The cross section of the solid taken perpendicular to the x-axis is a rectangle whose height is 4 times the length of its base in region R. Write, but do not evaluate, an integral expression that gives the volume of the solid.

66. AB Calculus – Step-by-Step (Calculators Allowed) **Name** ________________________________

Let $f(x)=e^{2-x^2}$. Let R be the region in the 2nd quadrant bounded by $f(x)$, the y-axis and the line $y=x+5$. Let S be the region in the 1st quadrant bounded by $f(x)$, the y-axis and the lines $y=x+3$ and $y=x+5$.



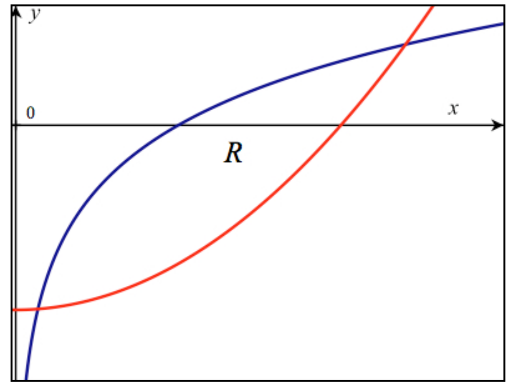
a. Find the area of R.

b. Find the area of S.

c. Let k be the maximum value of $f(x)$. Write, but do not evaluate the integral expression that gives the volume of the solid generated when region R is rotated about the line $y=k$.

67. AB Calculus – Step-by-Step (Calculators Allowed) **Name** ______________________________

Let R be the region bounded by the curves $y = \ln x^2$ and $y = x^2 - 4$ to the right of the y-axis as shown in the figure to the right.



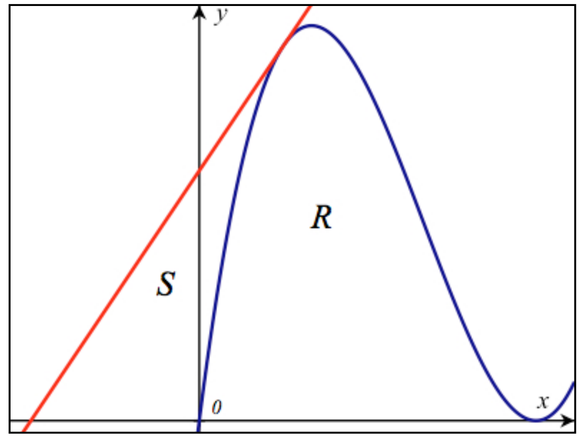
a. Find the area of R.

b. Find the volume generated when region R is rotated about the line $y = -4$.

c. Write, but do not evaluate the integral expression that gives the volume of the solid generated when region R is rotated about the y-axis.

68. AB Calculus – Step-by-Step **Name** ______________________________

Let f be the function given by $f(x) = x^3 - 16x^2 + 64x$ and let line l be the line tangent to the graph of f at $x = 2$, as shown in the figure to the right. Let R be the region bounded by the graph of f and the x-axis and let S be the region bounded by the graph of f, line l, and the x-axis.



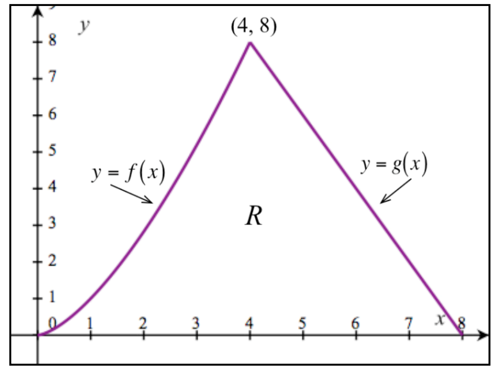
a. Find the equation of line l.

b. Find area of region S.

c. Region R is the base of a solid. The cross section of the solid taken perpendicular to the x-axis are semicircles. Write, but do not evaluate, an integral expression that gives the volume of the solid.

69. AB Calculus – Step-by-Step **Name** ________________________________

The functions f and g are given by $f(x)=\sqrt{x^3}$ and $g(x)=16-2x$. Let R be the region bounded by the x-axis and the graphs of f and g, as shown in the figure to the right.



a. Find the area of R.

b. The region of R from $x = 0$ to $x = 4$ is rotated about the line $x = 4$. Write, but do not evaluate, an integral expression the volume of the solid.

c. The region R is the base of a solid. For each y, the cross section of the solid taken perpendicular to the y-axis is a solid whose base lies in R and whose height is $h(y)$. Write, but do not evaluate, an integral expression of the volume of the solid.