70. AB Calculus – Step-by-Step **Name** ________________________________

Consider the differential equation $\frac{dy}{dx} = \frac{x+1}{y}$.

a. Let $f(x)$ be the particular solution to the differential equation passing through the point (2, 1). Write an equation for the line tangent to the graph of f at (2, 1) and use it to approximate the root of $f(x)$.

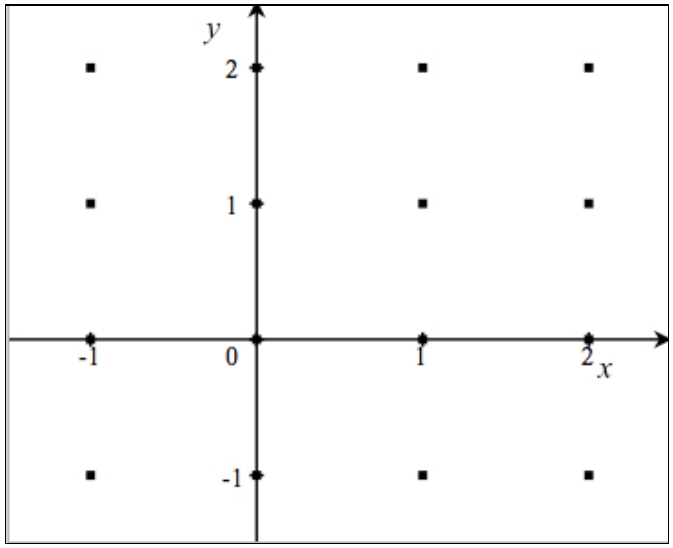
b. Find the particular solution to the differential equation with initial condition $f(-1) = -1$.

c. Let $y = f(x)$ be a particular solution to the differential equation with the condition $f(-1) = -1$. Does f have a relative maximum, relative minimum, or neither at $x = -1$? Justify your answer.

71. AB Calculus – Step-by-Step **Name** ________________________________

Consider the differential equation $\frac{dy}{dx} = \frac{y}{x-1}$.

a. On the axes provided, sketch a slope field for the given differential equation at the 16 points indicated and for $-1 < x < 2$, sketch the solution curve through (0, 1).



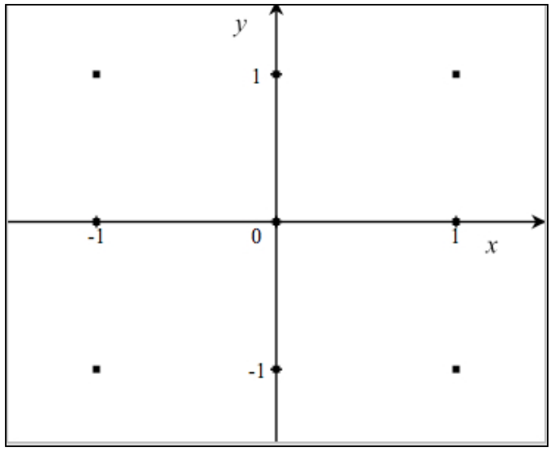
b. Describe all points in the *xy*-plane for which $\frac{dy}{dx} = -1$.

c. Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(0) = 4$.

72. AB Calculus – Step-by-Step **Name __________________________________**

Consider the differential equation $\frac{dy}{dx} = x - 2y + 1$.

a) On the axes provided, sketch a slope field for the given differential equation at the 9 points indicated.



b) Find $\frac{d^2y}{dx^2}$ in terms of x and y. Describe the region in the xy-plane in which all the solutions to the differential equation are concave down.

c) Let $y = f(x)$ be a particular solution to the differential equation with the condition $f(3) = 2$. Does f have a relative maximum, relative minimum, or neither at $x = 3$? Justify your answer.

d) Find the values of the constants m and b for which $y = mx + b$ is a solution to the differential equation.

73. AB Calculus – Step-by-Step **Name** ________________________________

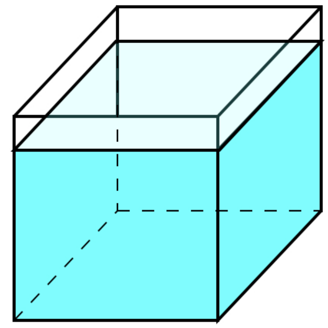
When a dam is built, the water from the river that is being dammed is blocked so that the dam can be built on a dry site. As the dam grows higher, residual water from the surrounding ground will collect behind the dam. At the Newton Dam, the water is 30 feet high at the time the dam is finished and the river is unblocked. The height H of the water after the river is unblocked satisfies the differential equation $\frac{dH}{dt} = \frac{1}{12}(H - 10)$, H measured in feet and t measured in months. This model is in effect for the next 2 years.

a. Use the line tangent to the graph of H at $t = 0$ to approximate the height of the water behind the dam after 1.5 months.

b. Find $\frac{d^2H}{dt^2}$ in terms of H. Use $\frac{d^2H}{dt^2}$ to determine whether your answer in part (a) is an underestimate or overestimate of the height of the water behind the dam at $t = 1.5$

c. Find the particular solution $H(t)$ to the differential equation $\frac{dH}{dt} = \frac{1}{12}(H - 10)$ with initial condition $H(0) = 30$ and use it to find an expression for the height of the water after 2 years.

74. AB Calculus – Step-by-Step **Name** ________________________________



A pool of water in the shape of a cube is shown in the figure to the right. Let s be the height of the water in the pool, where s is a function of tine t, measured in minutes. The volume V of the water in the pool is changing at the rate of $\frac{-s^{3/2}}{6}$ where s is the height of the water, measured in feet.

a. Show that $\frac{ds}{dt} = \frac{-1}{18\sqrt{s}}$.

b. Given that $s = 9$ feet at time $t = 0$, solve the differential equation $\frac{ds}{dt} = \frac{-1}{18\sqrt{s}}$ for s as a function of t.

c. At what time t is the pool empty?

75. AB Calculus – Step-by-Step (Calculators Allowed) **Name** ________________________________

The park service that administers a state park estimates that there are 495 deer in the park. They decide to remove deer according to the differential equation $\frac{dP}{dt} = -0.1P$.

a. Show that the solution to the differential equation $\frac{dP}{dt} = -0.1P$ is $P = 495e^{-.1t}$, where t is measured in years and P is the population of deer. Use it to find the deer population in 5 years to the nearest deer.

b. After this 5-year period, no human intervention is taken and the deer population grows again. From that time, the deer population increases directly proportional to $650 - P$, where the constant of proportionality is k. Find an equation for the deer population $P(t)$ in terms of t and k for this 5-year period.

c. Using the growth model from part b), 1 year later the deer population is 350. Find k.

d. Using the growth model from part b) and the value of k from part c), find $\lim_{t \to \infty} P(t)$.